

Diferansiyel Denklemler Dersi Vize Sınavı konularıyla ilgili Sorular

§ 1 (1—18): eğriler ailesinin diferansiyel denklemini yazınız.

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|---------------------------------|-------------------------------------|
| 1. $y = Cx^2 - x.$ | 2. $y = x^2 + Cx.$ |
| 3. $y = (x - C)^2.$ | 4. $(y - C)^2 = 2x.$ |
| 5. $(x - C)^2 + y^2 = 1.$ | 6. $x^2 + (y - C)^2 = 1.$ |
| 7. $2x^2 + Cy^2 = 1.$ | 8. $(y - C)^2 = \frac{1}{x}.$ |
| 9. $x^2 + 2x - (y - C)^2 = 2.$ | 10. $y = \operatorname{tg}(x + C).$ |
| 11. $Cx = \sin Cy.$ | 12. $Cy = \operatorname{tg} Cx.$ |
| 13. $x^2 = (C + y)e^y.$ | 14. $y^2 + 2Cxy + x^2 + 2x = 0.$ |
| 15. $y = A \cos(x + \varphi).$ | 16. $y = (C_1 + C_2x)e^x.$ |
| 17. $y = \frac{C_1}{x} + C_2x.$ | 18. $y^2 = C_1x^2 + C_2x.$ |

Problem 1.14-1.23 'de verilen diferansiyel denklemler için a) mertebeni, b) bilinmeyen fonksiyonu, c) bağımsız değişkeni belirleyiniz.

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|---|--|
| 1.14. $(y'')^2 - 3yy' + xy = 0$ | 1.15. $x^4y^{(4)} + xy''' = e^x$ |
| 1.16. $t^2\ddot{s} - t\dot{s} = 1 - \sin t$ | 1.17. $y^{(4)} + xy''' + x^2y'' - xy' + \sin y = 0$ |
| 1.18. $\frac{d^n x}{dy^n} = y^2 + 1$ | 1.19. $\left(\frac{d^2 r}{dy^2}\right)^2 + \frac{d^2 r}{dy^2} + y \frac{dr}{dy} = 0$ |
| 1.20. $\left(\frac{d^2 y}{dx^2}\right)^{3/2} + y = x$ | 1.21. $\frac{d^7 b}{dp^7} = 3p$ |
| 1.22. $\left(\frac{db}{dp}\right)^7 = 3p$ | 1.23. $y^{(6)} + 2y^4y^{(3)} + 5y^8 = e^x$ |

Problem 2.26-2.35 'de diferansiyel denklemler standart ve diferansiyel formda verilmişlerdir. Her iki form için denklemin tipini belirleyiniz.

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| 2.26. $y' = xy; \quad xy \, dx - dy = 0$ | 2.31. $y' = -\frac{2y}{x}; \quad 2xy \, dx + x^2 \, dy = 0$ |
| 2.27. $y' = xy; \quad x \, dx - \frac{1}{y} \, dy = 0$ | 2.32. $y' = \frac{xy^2}{x^2y + y^3}; \quad xy^2 \, dx - (x^2y + y^3) \, dy = 0$ |
| 2.28. $y' = xy + 1; \quad (xy + 1) \, dx - dy = 0$ | 2.33. $y' = \frac{-xy^2}{x^2y + y^2}; \quad xy^2 \, dx + (x^2y + y^2) \, dy = 0$ |
| 2.29. $y' = \frac{x^2}{y^2}; \quad \frac{x^2}{y^2} \, dx - dy = 0$ | 2.34. $y' = x^3y + xy^3; \quad (x^2 + y^2) \, dx - \frac{1}{xy} \, dy = 0$ |
| 2.30. $y' = \frac{x^2}{y^2}; \quad -x^2 \, dx + y^2 \, dy = 0$ | 2.35. $y' = 2xy + x; \quad (2xye^{-x^2} + xe^{-x^2}) \, dx - e^{-x^2} \, dy = 0$ |

(19–38): İzoklin yöntemiyle denklemin integral eğrilerini yaklaşık olarak çiziniz.

$$19. \quad y' = \frac{y-1}{x-1}.$$

$$20. \quad y' = \frac{y}{x+1}$$

$$21. \quad y' = \frac{1-x}{y-1}.$$

$$22. \quad y' = \frac{x+1}{1-y}.$$

$$23. \quad y' = \frac{1-y}{x}.$$

$$24. \quad y' = \frac{y}{1-x}.$$

$$25. \quad y' = (x-1)y.$$

$$26. \quad y' = x(y+1).$$

$$27. \quad y' = \frac{2x+y}{x-2y}.$$

$$28. \quad y' = \frac{y-2x}{2y+x}.$$

$$29. \quad y' = 2x + 2y + 1.$$

$$30. \quad y' = 2x - 2y - 1.$$

§ 2 (1–23): Değişkenlerine ayırarak denklemleri çözünüz

$$1. \quad y' = y^2 - y.$$

$$2. \quad (x^2 + x) y' - (2x + 1)y = 0.$$

$$3. \quad xy' \cos y + \sin y = \sin^2 y.$$

$$4. \quad y' \cos x + y(1 + y) \sin x = 0.$$

$$5. \quad 2xydx = (1 - x^2) dy.$$

$$6. \quad x^3 y dy = (x - 1) dx.$$

$$7. \quad yy' \cos x = (1 - y) \sin x.$$

$$8. \quad x(1 - y^2) y' = y(1 + y^2).$$

$$9. \quad (x^2 - 1) y dx = x(x^2 + 1) dy.$$

$$10. \quad x(y + 1) dy = (1 - y^2) dx.$$

$$11. \quad xy' + y^2 \left(\frac{1}{x} - 3x \right) = 0.$$

$$12. \quad (1 - x^2)^2 yy' + x = 0.$$

$$13. \quad (x + 1)y' + y(y + 1) = 0.$$

$$14. \quad (1 + y^2) y dx = x(1 + 2y^2) dy.$$

$$15. \quad x^2(x^2 + 4) y' = \cos^2 y.$$

$$16. \quad y' \operatorname{tg}^2 x - \operatorname{ctg} y = 0.$$

$$17. \quad (1 + \cos x)yy' = (1 + y^2) \sin x.$$

$$18. \quad ye^x dy + xe^{y^2} dx = 0.$$

$$19. \quad x(1 + y)y' + (\sqrt{x} + \ln x)(1 + y^2) = 0.$$

$$20. \quad (x - 1)yy' + (x^2 + 1)(y + 1)^2 = 0. \quad 21. \quad x^2 dx + (1 + x^6) \sqrt{1 - 2y} dy = 0.$$

$$22. \quad y' \sqrt{1 - x^4} + x(1 + e^y) = 0.$$

$$23. \quad y' \frac{\sin^2 x}{\cos x} + e^{-y} \sqrt{1 + e^y} = 0.$$

§ 2 (24–27): Değişkenin lineer değiştirilmesi yöntemiyle değişkenlerine ayrılabilir şekle indirgeyerek çözünüz.

$$24. \quad (2x + y + 2)dx - (4x + 2y + 9)dy = 0.$$

$$25. \quad (4 - x - 2y)dx - 2(1 + x + 2y)dy = 0.$$

$$26. \quad (2y - x + 1)dx + (4y - 2x + 6)dy = 0.$$

$$27. \quad (y - 3x + 2)dx + (3x - y - 1)dy = 0.$$

§ 2 (28–39): Verilen başlangıç şartı sağlayan çözümleri bulunuz.

$$28. \quad 2y(1 + y^2) dx + x(3y^2 + y + 3) dy = 0, \quad y(1) = 1.$$

$$29. \quad x(2y - 1)y' + 4y^2 = 0, \quad y(-1) = -1.$$

$$30. \quad x(1 + y)y' = y^2, \quad y(1) = 1.$$

$$31. \quad 3x(x + 1)y' = (x + 2)y, \quad y(1) = -1.$$

32. $(y + 2)y' = \sin 2x, y(0) = 1.$
 33. $(e^x + 1)^2 y' + (e^{2x} - 1)y = 0, y(0) = \frac{1}{4}.$
 34. $(x^2 + x)y' - (x^2 + x + 1)y = 0, y(1) = \frac{e}{2}.$
 35. $(x^3 + x)y' - (3x^2 - 1)y = 0, y(-1) = -4.$
 36. $y' + 3y^2 = 3y, y(0) = \frac{1}{2}.$
 37. $y' = (y + y^4) \operatorname{th} x, y(0) = 1.$
 38. $xy' + y(1 + y) \sin x = 0, y(0) = 1.$
 39. $2y' = (y^2 - 2y)e^{x^2}, y(0) = 1.$

§ 2 (57—78): Denklemlerin homojen olduğunu gösteriniz ve çözünüz.

57. $xy' = y \left(1 + \ln \frac{y}{x}\right).$ 58. $xy' = \frac{x^2 + y^2}{x + y}.$
 59. $xdy = (y + \sqrt{x^2 + y^2}) dx.$ 60. $xydx = (x^2 - y^2) dy.$
 61. $xdy = (y - \sqrt{x^2 + y^2}) dx.$ 62. $(x + 2y)dx + ydy = 0.$
 63. $(xy + y^2)y' = y^2.$ 64. $(2x + y)y' = x + 2y.$
 65. $(x^3 + y^3)y' = x^2y.$ 66. $(x + 2y)y' + y = 0.$
 67. $(y - x)y' = x + y.$ 68. $x^2y' = 2y^2 - xy.$
 69. $x^2y' = y^2 + 2xy.$ 70. $2xyy' + x^2 - y^2 = 0.$
 71. $(3xdy - ydx)(x^2 + y^2) + x^2ydy - xy^2dx = 0.$
 72. $(x + y + 1)dx + (x - y + 3)dy = 0.$
 73. $(2x - y - 2)dx + (x + y - 4)dy = 0.$
 74. $(x + 2y - 5)dx + (y - x - 4)dy = 0.$
 75. $(x - 1)y' + 3x + 2y + 3 = 0.$ 76. $(x + y - 2)y' + x - y = 0.$
 77. $(2x + y - 3)y' + y + 1 = 0.$ 78. $(x + 2y)y' + 2x + 5y - 1 = 0.$

§ 2 (79—83): Verilen başlangıç şartı sağlayan çözümleri bulunuz.

79. $y' = \frac{2x + y}{x - 2y}, y(1) = 0.$ 80. $y' = \frac{y - 2x}{x + 2y}; y(1) = 0.$
 81. $(y^2 - 3x^2)y' + xy = 0, y(1) = \sqrt{6}.$
 82. $xyy' = (x - 2y)^2, y(1) = 2.$ 83. $(x - y)^2y' = 4xy, y(-1) = 2.$

§ 2 (84—87): $y = z^m$ değişken deşistirmesi yaparak denklemleri homojen şekle indirgeyiniz ve çözünüz.

84. $(4x^2 + y^4)dy - 2xydx = 0.$ 85. $(3x^2y^2 + 1)y' + 3xy^3 = 0.$
 86. $y' = 4x^2 - \frac{y^2}{x^4}.$ 87. $y' = x + \frac{x^3}{y}.$

§ 3 (1—31): Lineer denklemlerin genel çözümlerini bulunuz.

1. $y' + y = 2e^x$.

2. $xy' = y - 2x^2$.

3. $y^2 dx + (xy - 1) dy = 0$.

4. $2y dx + \left(\frac{3}{y} - x\right) dy = 0$.

5. $x(4 - x^2) y' = 2x^2 y + 1$.

6. $xy' = x^2 + y$.

7. $y' = \frac{y}{x} - x$.

8. $(x + y) dx = x dy$.

9. $2x^3 y' = 2x^2 y - 3$.

10. $y dx - (x + y^2) dy = 0$.

11. $y dx = (3x - y^2) dy$.

12. $y' = y + 2xe^x$.

13. $(x + y^2 \cos y) dy = y dx$.

14. $xy' = x^2 + y - \frac{1}{x}$.

15. $y' = \frac{y}{x} - 2x^2$.

16. $x^4 dy = (2 - x^3 y) dx$.

17. $y dx = (2y - x) dy$.

18. $dx = (2x + e^y) dy$.

19. $x^3 y' + 2x^2 y = 2 \ln x$.

20. $(\sin x - 1)y' + y \cos x = \sin x$.

21. $y' + 2xy = 2(1 + 2x^2)$.

22. $xy' - 2y = 2x^4$.

23. $x^4 y' + 2x^3 y = 1$.

24. $x^2 y' + 2xy = 1$.

25. $xy' - 3y = 4x^2$.

26. $x^3 y' + x^2 y = x^2 - 1$.

27. $4y' + 12x^2 y = 3x^2$.

28. $xy' + (1 + x^2)y + x = 0$.

29. $x(y - \sqrt{1 + x^2}) dx + (1 + x^2) dy = 0$.

30. $y' + y \operatorname{tg} x = e^x \cos x$.

31. $(1 + y^2) dx + (xy - y^3) dy = 0$.

§ 3 (32—35) yapay yöntem kullanarak kısa yoldan çözünüz.

32. $xy' - y = x^2$.

33. $(y + x^3 \cos x) dx - x dy = 0$.

34. $x^2 y' + xy + 1 = 0$.

35. $(1 + y^2) dx + (2xy - 1) dy = 0$.

§ 3 (36—45): Denklemin verilen başlangıç şartı sağlayan çözümleri bulunuz.

36. $xy' + 2y = 3x, y(-1) = 1$.

37. $x^2 y' = 5xy + 6, y(1) = 1$.

38. $xy' = 7y + x^4, y(1) = -\frac{1}{3}$.

39. $xy' = 5y + 3x^2, y(-1) = -1$.

40. $(1 + x^2) y' = 2xy - 2x, y(0) = 2$.

41. $y' - y \operatorname{tg} x = \sin x, y(0) = 0$.

42. $(x^2 + x) y' - (x^2 + x + 1) y + x^3 = 0, y(1) = 1$.

43. $xy' = 3y + 2x^5, y(-1) = 1$.

44. $xy' - 2y = 2x^4, y(1) = 1$.

45. $x^2 y' + y = 4, y(-1) = 5$.

§ 3 (47—72): Bernoulli denklemlerini çözünüz.

47. $4xy' + (4x + 1)y^2 - 4y = 0$.

48. $2xy' + 2y = x^2 y^2$.

49. $y' = xy^2 + \frac{y}{x}$.

50. $2xy' = 3y - 4xy^3$.

§ 3 51. $xy' - y + 2xy^2 \ln x = 0$.

52. $2xy' + 2xy^3 = y$.

53. $2x^2y' + xy = 2y^3$.

54. $xy' + 4y = 3xy^2$.

55. $y' - \frac{y}{x} = y^2$.

56. $xy' = 2y - 4x^2y^2$.

57. $y' - y + 2xy^3 = 0$.

58. $xy' + 3xy^2 = 2y$.

59. $xy' - y + 4y^3 = 0$.

60. $y' + y \operatorname{tg} x + 4y^2 \sin x = 0$.

61. $xy' + 3y = 4x^2y^2$.

62. $xy' + 2xy^2 = 3y$.

63. $y(y+1)dx + (x+1)dy = 0$.

64. $y' \cos x + y \sin x + 3y^2 \cos x = 0$.

65. $5xy^4y' = y^5 + 4$.

66. $y(4xy^2 - 3)dx + 2xdy = 0$.

67. $8y' + 3x^2y(y^2 - 4) = 0$.

68. $ydx + (2x^2y - 3x)dy = 0$.

69. $3x^2dx - (x^3 + y + 1)dy = 0$.

70. $y^3dx + (x^3 \ln y - xy^2)dy = 0$.

71. $ydx + (4x^3 - x)dy = 0$.

72. $(y^2 - 1)dx - y[x + (y^2 - 1)\sqrt{x}]dy = 0$.

(82–95): Riccati denkleminin bir özel çözümünü seçerek genel çözümünü bulunuz.

82. $2x^2y' + x^2y^2 + 4 = 2xy$.

83. $x^2y' + x^2y^2 + 2xy = 2$.

84. $4y' = y^2 + \frac{4}{x^2}$.

85. $xy' = y^2 + 2(x+1)y + x^2 + x$.

86. $x^2y' = x^2y^2 + 3xy + 3$.

87. $x^2y' = y^2 + 2xy - 2x^2$.

88. $y' = y^2 - 2xy + x^2$.

89. $y' = y^2 - 2xy + x^2 - 3$.

90. $y' + e^{-x}y^2 + y = 3e^x$.

91. $y' - e^xy^2 + 3y = e^{-x}$.

92. $y' = y^2 - 2y \sin x + \cos x + \sin^2 x$.

93. $y' + y^2 - 2y \cos x + \sin x + \cos^2 x = 0$.

94. $x^2y' - 5xy + x^2y^2 + 8 = 0$.

95. $(3x^2 + 2y)(1+y)dx + (2x - x^3)dy = 0$.

§ 4 (1–18): Denklemlerin tam diferansiyel formu olduğunu gösteriniz ve çözünüz.

1. $(1 - 3x^2 - y)dx = (x - 3y^2)dy$.

2. $(y^2 - 2x^3)dx + 2xydy = 0$.

3. $[(x-y)^2 - x]dx + [y - (x-y)^2]dy = 0$.

4. $(y - \sin x)dx + (x + e^y)dy = 0$.

5. $(y - x)dx + (x + 2e^{2y})dy = 0$.

6. $(x + y)dy = (2e^{2x} - y)dx$.

7. $(y + \sin x)dx + (x + \cos y)dy = 0$.

8. $(2x + y^3)dx + 3xy^2dy = 0$.

9. $(y^2 - 2x) dx + (2xy - \sin y) dy = 0.$
10. $(y - 3x^2 + 1) dx + (x + \ln y) dy = 0.$
11. $(y^2 + \ln x) dx + (2xy - \ln y) dy = 0.$
12. $(e^x + y) dx + (x + 2y \cos y^2) dy = 0.$
13. $(1 + 3x^2 \ln y) dx + \left(3y^2 + \frac{x^3}{y}\right) dy = 0.$
14. $\left(2x - \frac{\sin^2 y}{x^2}\right) dx + \left(2y + \frac{\sin 2y}{x}\right) dy = 0.$
15. $\left(\frac{y}{x^2} + \frac{1}{y}\right) dx - \left(\frac{x}{y^2} + \frac{1}{x} + 2y\right) dy = 0.$
16. $e^{\frac{y}{x}} \left(1 - \frac{y}{x}\right) dx + \left(1 + e^{\frac{y}{x}}\right) dy = 0.$
17. $\frac{y}{x} dx + [1 + \ln(xy)] dy = 0, x > 0, y > 0.$
18. $\left(1 + \frac{2x}{y^3}\right) dx + \left(\frac{1}{y^2} - \frac{3x^2}{y^4}\right) dy = 0.$

§ 4 (19–60): İntegral çarpanını bularak veya uygun değişken değişirmesini yaparak denklemleri çözünüz.

19. $2xy dx + (y^2 - x^2) dy = 0.$
20. $2xy dx = (x^2 - 2y^3) dy.$
21. $(3\sqrt{x-y} - 2x) dx = (3\sqrt{x-y} - 2y) dy.$
22. $(y - 3x^2 y^3) dx - (x + x^3 y^2) dy = 0.$
23. $(2xy^2 + y) dx - (x^2 y + 2x) dy = 0.$
24. $y dx = (x - 2y^3) dy.$
25. $x^3 dy + 2(y - x^2) y dx = 0.$
26. $y^2 dx = (2y - e^x) dy.$
27. $x dy = y(1 - ye^x) dx.$
28. $y' = \frac{y}{x} + 2x^2.$
29. $x^2 dy = (xy + y^3) dx.$
30. $y^2 dx = x(2y - x) dy.$
31. $x^2 dy = (xy - 2x^2 y^2) dx.$
32. $x dy = y(2 + 3xy) dx.$
33. $\left(\frac{1}{x} - y\right) dx = \frac{1}{y} dy.$
34. $(2x^3 y + x^2 y^4) dx + (x^4 + x^3 y^3) dy = 0.$
35. $(3x^2 y + 7y^2) dx - (2x^3 + 5xy) dy = 0.$

36. $(2y - x^2y^2) dx = (x^3y - x) dy$.
37. $(4x^3y + 3y^2) dx - (2x^4 + xy) dy = 0$.
38. $(3xdy - ydx)(x^2 + y^2) + x^2ydy - xy^2dx = 0$.
39. $(3x^2 + 2y)(1 + y)dx + (2x - x^3) dy = 0$.
40. $(xy^3 + 2y) dx + (x - x^2y^2) dy = 0$.
41. $3xdy + ydx + xy^3(xdy + ydx) = 0$.
42. $xy^2dx + (x^2y - x) dy = 0$.
43. $x(x^2 + y^2) dy + y(ydx - xdy) = 0$.
44. $(xy^3 + y) dx + (2x + x^2y^2) dy = 0$.
45. $2x^4(xdx + ydy) + (x^2 + y^2)^2(xdy - 3ydx) = 0$.
46. $y(2ydx - xdy) + x^2(ydx + 2xdy) = 0$.
47. $(y^2 - 3x^2) dy + xydx = 0$.
48. $xdy - ydx = x\sqrt{x^2 + y^2} \cdot dx$.
49. $xdy - 2ydx + xy^2(2xdy + ydx) = 0$.
50. $x^2y^3dx + (x^3y^2 + x) dy = 0$.
51. $5xdy + ydx + xy^5(xdy - ydx) = 0$.
52. $xdy + ydx + xy^2(xdy - 5ydx) = 0, x > 0, y > 0$.
53. $2xdy + ydx + xy^3(xdy + 2ydx) = 0$.
54. $(y^2 + y) dx + \left(xy + 2x + \frac{1}{y}\right) dy = 0$.
55. $[(3x^2 + 2)y + 3x] dx + (2x - x^3) dy = 0$.
56. $xy' - y = 2x^3e^{-\frac{y}{x}}$.
57. $2xy^3dx + x^2y^2dy = (1 - y^2) dy$.
58. $(2xy^2 - y) dx + (y^2 \ln y + x - y) dy = 0$.
59. $x^2yy' + x^3 = (x^2 + y^2)^2$.
60. $4x^2y^2dx + x^3(2y - 1)dy = 0$.

§ 1 Probleminin cevapları:

1. $xy' - 2y = x$.
2. $xy' - y = x^2$.
3. $y'^2 = 4y$.
4. $2xy'^2 = 1$.
5. $y^2 (y'^2 + 1) = 1$.
6. $(1 - x^2) y'^2 = x^2$.
7. $(2x^2 - 1) y' = 2xy$.
8. $4x^3 y'^2 = 1$.
9. $(x^2 + 2x - 2) y'^2 = (x + 1)^2$.
10. $y' = 1 + y^2$.
11. $\frac{1}{y'} = \cos \frac{y\sqrt{y'^2 - 1}}{|xy'|}$.
12. $y' \cos^2 \frac{x\sqrt{y' - 1}}{|y|} = 1$.
13. $(x^2 + e^y) y' = 2x$.
14. $x(y^2 - x^2 - 2x) y' = y(y^2 - x^2)$.
15. $y'' + y = 0$.
16. $y'' - 2y' + y = 0$.
17. $x^2 y'' + xy' - y = 0$.
18. $x^2 (yy'' + y'^2) = y(2xy' - y)$.

§ 2 Probleminin cevapları:

1. $y(1 + Ce^x) = 1, y = 0$.
2. $y = C(x^2 + x)$.
3. $x \sin y = C(1 - \sin y)$.
4. $y = \frac{C \cos x}{1 - C \cos x}, y = -1$.
5. $(x^2 - 1)y = C$.
6. $y^2 = \frac{1}{x^2} - \frac{2}{x} + C, x = 0$.
7. $(y - 1)e^y = C \cos x$.
8. $x(y^2 + 1) = Cy, y = 0$.
9. $y^2 = C(x^2 - 1)^2 e^{x^2}, x = 0$.
10. $x(y - 1) = C, y = -1$.
11. $\frac{1}{y} + \frac{1}{x} = C - 3x, y = 0$.
12. $y^2 + \frac{1}{1 - x^2} = C$.
13. $(x + 1)y = C(y + 1), y = -1$.
14. $x^2 = Cy^2(1 + y^2), y = 0$.
15. $\operatorname{tg} y = -\frac{1}{4x} - \frac{1}{8} \operatorname{arctg} \frac{x}{2} + C, y = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$.
16. $\ln |\cos y| - \operatorname{ctg} x - x = C, y = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$.
17. $(1 + y^2)(1 + \cos x)^2 = C$.
18. $e^{-x^2} + 2(x + 1)e^{-x} = C$.
19. $4\sqrt{x} + \ln^2 x + 2 \operatorname{arctg} y + \ln(1 + y^2) = C$.
20. $\frac{x^2}{2} + x + \ln(x - 1)^2 + \ln|y + 1| + \frac{1}{1 + y} = C, y = -1$.
21. $\operatorname{arctg} x - \sqrt{(1 - 2x)^3} = C$.
22. $\frac{1}{2} \arcsin x^2 + y - \ln(1 + e^y) = C$.
23. $2\sqrt{1 + e^y} - \frac{1}{\sin x} = C$.
24. $y + 2x + 4 = Ce^{x-2y}$.
25. $(x + 2y)^2 - 3x + 4y = C$.
26. $2y - x + 2 = Ce^{x+2y}$.
27. $2y - 6x + 1 = Ce^{2y-2x}$.
28. $\ln x^2 + 3 \ln y + \operatorname{arctg} y = \frac{\pi}{4}$.

29. $\ln(x^4 y^2) + \frac{1}{y} + 1 = 0$.
31. $2x^2 + (x+1)y^3 = 0$.
33. $y = \frac{e^x}{(1+e^x)^2}$.
35. $y = \frac{(1+x^2)^2}{x}$.
37. $y\sqrt[3]{2 - \operatorname{ch}^3 x} = \operatorname{ch} x$.
39. $y(1 + e^{\int_0^x e^{t^2} dt}) = 2$.
41. $x^2 + Cy^2 = C$.
43. $Ce^{\frac{1}{2}y^2 + 2y} = \cos x$.
45. $e^{2y - \frac{1}{2}y^2} \cdot \sin x = C$.
47. $x^2 + \ln(x^2) + 4(y - e^{-y}) = C$.
49. $(y-1)e^{y + \frac{1}{2}x^2} = C$.
51. $x^2 + y^2 = 2a^2 \ln Cx$.
53. $f' \cdot r' + f \cdot r = 0, r = Ce^{-\varphi}$.
57. $y = xe^{Cx}$.
59. $y + \sqrt{x^2 + y^2} = Cx^2, x = 0$.
61. $y + \sqrt{x^2 + y^2} = C, x = 0$.
63. $x = Cy + y \ln |y|, y = 0$.
65. $y = Ce^{\frac{x^3}{3y^3}}$.
67. $x^2 + 2xy - y^2 = C$.
69. $x^2 + xy = Cy, y = 0$.
71. $\ln \left| \frac{y^3}{x} \right| + \operatorname{arctg} \frac{y}{x} = C, x = 0, y = 0$.
73. $y^2 + 2x^2 - 4y - 8x + 12 = Ce^{\sqrt{2} \operatorname{arctg} \frac{y-2}{(x-2)\sqrt{2}}}$.
74. $x^2 + y^2 + xy - x - 5y + 7 = Ce^{2\sqrt{3} \operatorname{arctg} \frac{x+2y+1}{(x+1)\sqrt{3}}}$.
75. $x + y + 2 = C(x-1)e^{\frac{3}{x-1}}, x = 1$.
76. $x^2 + y^2 - 2x - 2y + 2 = Ce^{-2 \operatorname{arctg} \frac{y-1}{x-1}}$.
77. $(y+1)(x-2)^3 = C(3x+y-5)$.
30. $\ln \frac{x}{y} + \frac{1}{y} = 1$.
32. $y^2 + 4y + \cos 2x = 6$.
34. $y = \frac{xe^x}{x+1}$.
36. $y = \frac{1}{1 + e^{-3x}}$.
38. $y(2e^{\int_0^x \frac{\sin t}{t} dt} - 1) = 1$.
40. $x^2 = Ce^{y^2 - 2x}$.
42. $xe^{y^2 + \frac{1}{3}y^3} = C$.
44. $x = \frac{1}{2}y^2 - \frac{1}{3}y^3 + C$.
46. $e^y - y + x = C$.
48. $(y+2)^2 e^{x-y} = C$.
50. $(2x+y)^2 - 6x + 2y = C$.
52. $x^2 + y^2 + 2b^2 \ln Cy = 0$.
54. $f \cdot r' + r = 0, r = C \sin \varphi$.
58. $Cx = (y-x)^2 e^{\frac{y}{x}}$.
60. $y = Ce^{-\frac{x^2}{2y^2}}$.
62. $x + y = Ce^{-\frac{x}{x+y}}$.
64. $(y-x)^3 = C(y+x), y = -x$.
66. $y^2 + xy = C$.
68. $x - y = Cx^2 y, y = 0$.
70. $x^2 + y^2 = Cx$.
72. $x^2 + 2xy - y^2 + 2x + 6y = C$.
78. $x^2 + 2y^2 + 3xy + 2y = C \frac{x+2y}{x+y+1}$.

79. $x^2 + y^2 = e^{\operatorname{arctg} \frac{y}{x}}.$ 80. $(x^2 + y^2) e^{\operatorname{arctg} \frac{y}{x}} = 1.$
81. $y^3 = 3\sqrt{6} \cdot \sqrt{y^2 - 2x^2}.$ 82. $3y - x = 5(y - x) \cdot e^{\frac{2-y}{2}}.$
83. $2(y+x)^3(y-3x) = 5y.$ 84. $y^2 = Ce^{\frac{2x^2}{y^4}}.$
85. $y^2 e^{3x^2 y^2} = C.$ 86. $x^5 (y - x^3) = C (y + 4x^3).$
87. $(y - x^2)^2 (2y + x^2) = C.$
88. a) $y = \frac{x^4}{4} + 1$, б) $y = x^2$, $y = \frac{x^4 + 1}{2}.$
89. $y = C (x^2 + y^2).$ 90. $(x - 2y)y' = x + 2y.$
91. a) $x (3y^2 - x^2) dx = y (3x^2 - y^2) dy$, б) $y^2 - x^2 = C (x^2 + y^2)^2.$
92. $y' = \frac{x^2 + 2xy - y^2}{x^2 - 2xy - y^2}.$

§ 3 Problemlerinin cevapları:

1. $y = Ce^{-x} + e^x.$ 2. $y = Cx - 2x^2.$
3. $xy = C + \ln |y|$, $y = 0.$ 4. $x = C\sqrt{y} + \frac{1}{y}.$
5. $y = \frac{C + \ln |x|}{4 - x^2}.$ 6. $y = Cx + x^2.$
7. $y = Cx - x^2.$ 8. $y = Cx + x \ln |x|$, $x = 0.$
9. $y = Cx + \frac{1}{2x^2}.$ 10. $x = Cy + y^2$, $y = 0.$
11. $x = Cy^3 + y^2$, $y = 0.$ 12. $y = (C + x^2) e^x.$
13. $x = y(C + \sin y)$, $y = 0.$ 14. $y = Cx + x^2 + \frac{1}{2x}.$
15. $y = Cx - x^3.$ 16. $y = \frac{C}{x} - \frac{1}{x^3}$, $x = 0.$
17. $xy - y^2 = C.$ 18. $x = Ce^{2y} - e^y.$
19. $y = \frac{C}{x^2} + \frac{\ln^2 x}{x^2}.$ 20. $y = \frac{C - \cos x}{\sin x - 1}.$
21. $y = Ce^{x^2} + 2x.$ 22. $y = \frac{C}{x^2} + x^4.$
23. $y = \frac{C}{x^2} + \frac{x}{3}.$ 24. $y = \frac{C}{x^2} + \frac{1}{x}.$
25. $y = Cx^3 - 4x^2.$ 26. $y = \frac{C}{x} + 1 + \frac{1}{x^2}.$
27. $y = Ce^{-x^3} + \frac{1}{4}.$ 28. $y = \frac{C}{x} e^{-\frac{x^2}{2}} - \frac{1}{x}.$

29. $y = \frac{C + x^2}{2\sqrt{1 + x^2}}.$ 30. $y = (C + e^x) \cos x.$
31. $(3x - y^2 + 2) \sqrt{1 + y^2} = C.$ 32. $y = Cx + x^2.$
33. $y = Cx + x^2 \sin x + x \cos x, x = 0.$ 34. $xy + \ln|x| = C.$
35. $x(1 + y^2) = y + C.$ 36. $y = x + \frac{2}{x^2}.$
37. $y = 2x^5 - \frac{1}{x}.$ 38. $y = -\frac{1}{3}x^4.$
39. $y = -x^2.$ 40. $y = x^2 + 2.$
41. $y = \frac{1 - \cos 2x}{\cos x}.$ 42. $y = x.$
43. $y = -2x^3 + x^5.$ 44. $y = x^4.$
45. $y = e^{-\frac{x+1}{x}} + 4.$ 46. $(x + y + 1)e^{-y} = C.$
47. $\frac{1}{y} = \frac{C}{x} + \frac{2x + 1}{4}.$ 48. $\frac{1}{y} = Cx - \frac{1}{2}x^2.$
49. $\frac{1}{y} = \frac{C}{x} - \frac{1}{3}x^2.$ 50. $\frac{1}{y^2} = \frac{C}{x^3} + x.$
51. $\frac{1}{y} = \frac{C}{x} + x \ln x - \frac{1}{2}x.$ 52. $\frac{1}{y^2} = \frac{C}{x} + x.$
53. $\frac{1}{y^2} = Cx + \frac{1}{x}.$ 54. $\frac{1}{y} = Cx^4 + x.$
55. $\frac{1}{y} = \frac{C}{x} - \frac{x}{2}.$ 56. $\frac{1}{y} = \frac{C}{x^2} + x^2.$
57. $\frac{1}{y^2} = Ce^{-2x} + 2x - 1.$ 58. $\frac{1}{y} = \frac{C}{x^2} + x.$
59. $\frac{1}{y^2} = \frac{C}{x^2} + 4.$ 60. $\frac{1}{y} = \frac{C}{\cos x} - 2 \cos x.$
61. $\frac{1}{y} = Cx^3 + 4x^2.$ 62. $\frac{1}{y} = \frac{C}{x^3} + \frac{x}{2}.$
63. $(x + 1 - C)y = C.$ 64. $(3 \sin x + C)y = \cos x, y = 0.$
65. $y^5 + 4 = Cx.$ 66. $y^2(x^4 + C) = x^3, y = 0.$
67. $y^2(e^{-x^3} + C) = 4C.$ 68. $x(y^4 + 1) = 2Cy^3, y = 0.$
69. $x^3 + y + 2 = Ce^y.$ 70. $y^2 = x^2(\ln^2 y + C), x = 0.$
71. $x^2(4y^2 + C) = y^2, x = 0.$
72. $3\sqrt{x} = y^2 - 1 + C|y^2 - 1|^{\frac{1}{4}}, x = 0, y = \pm 1.$
73. $y = \sqrt{2x^{\frac{3}{2}} - x^2}.$ 74. $x = 4y^4 \ln^2 y.$

$$75. x(2 - y^2) = 1.$$

$$77. x^2 e^{\frac{x^2}{y^2}} = C, y = 0.$$

$$79. y(2x^2 + x + C) = 4x, y = 0.$$

$$81. y = \cos^2 x.$$

$$83. y = \frac{1}{x} + \frac{16}{Ce^{4x} - 4x - 1}.$$

$$85. y = -x + \frac{2Cx^2}{1 - Cx^2}.$$

$$87. y = x + \frac{3x^4}{C - x^3}.$$

$$89. y = x - 2 + \frac{4}{1 + Ce^{4x}}.$$

$$91. y = e^{-x} + \frac{e^{-x}}{C - x}.$$

$$93. y = \cos x + \frac{1}{C - x}.$$

$$95. C(1 + y) = (2y + x^2)x.$$

$$96. y = \int_{+\infty}^x e^{k(x-t)} f(t) dt, k > 0; y = \int_{-\infty}^x e^{k(x-t)} f(t) dt, k < 0.$$

$$98. \frac{\beta}{\alpha}.$$

$$100. -\frac{B}{A}.$$

$$101. 0.$$

$$76. y(Cx^2 + 1) = 2Cx.$$

$$78. x(Cy^3 + 1) = 3Cy, y = 0.$$

$$80. y = \sin^2 x.$$

$$82. y = \frac{2}{x} + \frac{2}{Cx + x \ln |x|}.$$

$$84. y = -\frac{2}{x} + \frac{1}{x(C - \ln |x|)}.$$

$$86. y = -\frac{1}{x} + \frac{2x}{C - x^2}.$$

$$88. y = x + 1 + \frac{2Ce^{2x}}{1 - Ce^{2x}}.$$

$$90. y = e^x + \frac{4}{Ce^{3x} - e^{-x}}.$$

$$92. y = \sin x + \frac{1}{C - x}.$$

$$94. y = \frac{2}{x} + \frac{2x}{x^2 + C}.$$

§ 4 Problemlerinin cevabı:

$$1. y^3 - x^3 - xy + x = C.$$

$$2. y^2 = \frac{C}{x} + \frac{x^3}{2}, x = 0.$$

$$3. 2(y - x)^3 = 3(y^2 - x^2) + C.$$

$$4. xy + \cos x + e^y = C.$$

$$5. x^2 + C = 2xy + 2e^{2y}.$$

$$6. y^2 + 2xy = 2e^{2x} + C.$$

$$7. xy - \cos x + \sin y = C.$$

$$8. y^3 = \frac{C}{x} - x, x = 0.$$

$$9. xy^2 - x^2 + \cos y = C.$$

$$10. x(y + 1) - x^3 + y(\ln y - 1) = C.$$

$$11. xy^2 + x(\ln x - 1) + y(1 - \ln y) = C. \quad 12. e^x + xy + \sin y^2 = C.$$

$$13. x^3 \ln y + x + y^3 = C.$$

$$14. x^2 + y^2 + \frac{\sin^2 y}{x} = C.$$

$$15. \frac{x}{y} - \frac{y}{x} - y^2 = C.$$

$$16. y + xe^{\frac{y}{x}} = C.$$

$$17. y \ln(xy) = C.$$

$$18. x + \frac{x^2}{y^3} - \frac{1}{y} = C.$$

$$19. x^2 + y^2 = Cy, y = 0.$$

$$20. x^2 = Cy - y^3, y = 0.$$

21. $x^2 - y^2 = 2(x - y)^{\frac{3}{2}} + C$. 22. $x - x^3y^2 = Cy, y = 0$.
23. $x + x^2y^2 = Cy^2, y = 0$. 24. $x = Cy - y^2, y = 0$.
25. $x^2 - 2y \ln |x| = Cy, x = 0, y = 0$. 26. $y^2e^{-x} = y + C$.
27. $x = ye^x + Cy, x = 0, y = 0$. 28. $y = Cx + x^3$.
29. $\frac{x^2}{y^2} + 2x = C, y = 0$. 30. $y^2 - xy = Cx, x = 0$.
31. $x - x^2y = Cy, y = 0$. 32. $x^3y + x^2 = Cy, y = 0$.
33. $\frac{x^2}{2} - \frac{x}{y} = C$. 34. $x^7y^4(7x + 4y^3) = C$.
35. $\frac{x^9}{y^6} + \frac{3x^7}{y^5} = C, y = 0$. 36. $xye^{\frac{1}{x^2y}} = C, x = 0, y = 0$.
37. $x^2 = Cy \cdot e^{\frac{y}{2x^3}}, x = 0, y = 0$. 38. $\ln \left| \frac{y^3}{x} \right| + \operatorname{arctg} \frac{y}{x} = C, x = 0, y = 0$.
39. $\frac{x^3 + 2xy}{1 + y} = C, y = -1$. 40. $\frac{1}{x^2y} + \frac{y}{x} = C, x = 0, y = 0$.
41. $xy = Ce^{\frac{1}{xy^3}}, x = 0, y = 0$. 42. $xy - \ln |y| = C, x = 0, y = 0$.
43. $xe^y = C\sqrt{x^2 + y^2}, y = 0$. 44. $xy = Ce^{\frac{1}{xy^2}}, x = 0, y = 0$.
45. $\frac{1}{x^2 + y^2} = \frac{y}{x^3} + C, x = 0$. 46. $xy^2 = Ce^{\frac{y}{x^2}}, x = 0$.
47. $\frac{2}{x} - \frac{3}{y} = Cy, y = 0$. 48. $xe^x = C(y + \sqrt{x^2 + y^2})$.
49. $ye^{xy^2} = Cx^2, x = 0$. 50. $y^2e^{x^2y^2} = C, x = 0$.
51. $y = Cxe^{\frac{1}{xy^3}}, x = 0, y = 0$. 52. $(xy)^{11} = C(11xy^2 - 1)^6$.
53. $x^5y^{10} = C(1 + 5xy^3)^3$. 54. $Cy(xy + 1) = y + 1$.
55. $(4xy + 3)C = (2 - x^2)^2, x = 0$. 56. $e^{\frac{y}{x}} = x^2 + C$.
57. $y = C(x^2 + 1)y^2 + C$. 58. $x^2 + y(\ln y - 1) - \ln y - \frac{x}{y} = C$.
59. $(x^2 + y^2)(Cx + 2) = x$. 60. $x^4y^2e^{\frac{1}{y}} = C, y = 0$.
61. $y = x + \sqrt{x^2 + 8 + \frac{2}{x}}$.