İMÖ3011 Analiz III. Vize sınavı konuları

1-6 Durumlarında dizinin ilk on ardışık terimlerini yazınız:

1. $a_n = \dfrac{1-n}{n^2}$

2. $a_n = \dfrac{1}{n!}$

3. $a_n = \dfrac{(-1)^{n+1}}{2n-1}$

4. $a_n = 2 + (-1)^n$

5. $a_n = \dfrac{2^n}{2^{n+1}}$

6. $a_n = \dfrac{2^n - 1}{2^n}$

7–12 Örneklerinde a_1, a_2, a_3 ve a_4 terimlerini bulunuz, genel terim için formül yazınız:

7. $a_1 = 1, \quad a_{n+1} = a_n + (1/2^n)$

8. $a_1 = 1, \quad a_{n+1} = a_n/(n+1)$

9. $a_1 = 2, \quad a_{n+1} = (-1)^{n+1} a_n/2$

10. $a_1 = -2, \quad a_{n+1} = n a_n/(n+1)$

11. $a_1 = a_2 = 1, \quad a_{n+2} = a_{n+1} + a_n$

12. $a_1 = 2, \quad a_2 = -1, \quad a_{n+2} = a_{n+1}/a_n$

13–26 Örneklerinde dizinin genel terimi için formül yazınız:

13. Dizi: $1, -1, 1, -1, 1, \ldots$

14. Dizi: $-1, 1, -1, 1, -1, \ldots$

15. Dizi: $1, -4, 9, -16, 25, \ldots$

16. Dizi: $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, \ldots$

17. $\frac{1}{9}, \frac{2}{12}, \frac{2^2}{15}, \frac{2^3}{18}, \frac{2^4}{21}, \ldots$

18. $-\frac{3}{2}, -\frac{1}{6}, \frac{1}{12}, \frac{3}{20}, \frac{5}{30}, \ldots$

19. Dizi: $0, 3, 8, 15, 24, \ldots$

20. Dizi: $-3, -2, -1, 0, 1, \ldots$

21. Dizi: $1, 5, 9, 13, 17, \ldots$

22. Dizi: $2, 6, 10, 14, 18, \ldots$

23. $\frac{5}{1}, \frac{8}{2}, \frac{11}{6}, \frac{14}{24}, \frac{17}{120}, \ldots$

24. $\frac{1}{25}, \frac{8}{125}, \frac{27}{625}, \frac{64}{3125}, \frac{125}{15{,}625}, \ldots$

25. Dizi: $1, 0, 1, 0, 1, \ldots$

26. Dizi: $0, 1, 1, 2, 2, 3, 3, 4, \ldots$

27–88 de verilmiş $\{a_n\}$ dizilerinden hangisinin yakınsak, hangisinin ıraksak olduğunu belirleyiniz ve yakınsak olanların limitini bulunuz:

27. $a_n = 2 + (0.1)^n$

28. $a_n = \dfrac{n + (-1)^n}{n}$

29. $a_n = \dfrac{1-2n}{1+2n}$

30. $a_n = \dfrac{2n+1}{1-3\sqrt{n}}$

31. $a_n = \dfrac{1-5n^4}{n^4+8n^3}$

32. $a_n = \dfrac{n+3}{n^2+5n+6}$

33. $a_n = \dfrac{n^2-2n+1}{n-1}$

34. $a_n = \dfrac{1-n^3}{70-4n^2}$

43. $a_n = \sin\left(\dfrac{\pi}{2} + \dfrac{1}{n}\right)$

44. $a_n = n\pi \cos(n\pi)$

45. $a_n = \dfrac{\sin n}{n}$

46. $a_n = \dfrac{\sin^2 n}{2^n}$

47. $a_n = \dfrac{n}{2^n}$

48. $a_n = \dfrac{3^n}{n^3}$

49. $a_n = \dfrac{\ln(n+1)}{\sqrt{n}}$

50. $a_n = \dfrac{\ln n}{\ln 2n}$

35. $a_n = 1 + (-1)^n$

36. $a_n = (-1)^n\left(1 - \frac{1}{n}\right)$

37. $a_n = \left(\frac{n+1}{2n}\right)\left(1 - \frac{1}{n}\right)$

38. $a_n = \left(2 - \frac{1}{2^n}\right)\left(3 + \frac{1}{2^n}\right)$

39. $a_n = \frac{(-1)^{n+1}}{2n-1}$

40. $a_n = \left(-\frac{1}{2}\right)^n$

41. $a_n = \sqrt{\frac{2n}{n+1}}$

42. $a_n = \frac{1}{(0.9)^n}$

51. $a_n = 8^{1/n}$

52. $a_n = (0.03)^{1/n}$

53. $a_n = \left(1 + \frac{7}{n}\right)^n$

54. $a_n = \left(1 - \frac{1}{n}\right)^n$

55. $a_n = \sqrt[n]{10n}$

56. $a_n = \sqrt[n]{n^2}$

57. $a_n = \left(\frac{3}{n}\right)^{1/n}$

58. $a_n = (n+4)^{1/(n+4)}$

59. $a_n = \frac{\ln n}{n^{1/n}}$

60. $a_n = \ln n - \ln(n+1)$

61. $a_n = \sqrt[n]{4^n n}$

62. $a_n = \sqrt[n]{3^{2n+1}}$

63. $a_n = \frac{n!}{n^n}$ (*Hint:* Compare with $1/n$.)

64. $a_n = \frac{(-4)^n}{n!}$

65. $a_n = \frac{n!}{10^{6n}}$

66. $a_n = \frac{n!}{2^n \cdot 3^n}$

67. $a_n = \left(\frac{1}{n}\right)^{1/(\ln n)}$

68. $a_n = \ln\left(1 + \frac{1}{n}\right)^n$

69. $a_n = \left(\frac{3n+1}{3n-1}\right)^n$

70. $a_n = \left(\frac{n}{n+1}\right)^n$

71. $a_n = \left(\frac{x^n}{2n+1}\right)^{1/n}, \quad x > 0$

72. $a_n = \left(1 - \frac{1}{n^2}\right)^n$

73. $a_n = \frac{3^n \cdot 6^n}{2^{-n} \cdot n!}$

74. $a_n = \frac{(10/11)^n}{(9/10)^n + (11/12)^n}$

75. $a_n = \tanh n$

76. $a_n = \sinh(\ln n)$

77. $a_n = \frac{n^2}{2n-1}\sin\frac{1}{n}$

78. $a_n = n\left(1 - \cos\frac{1}{n}\right)$

79. $a_n = \sqrt{n}\sin\frac{1}{\sqrt{n}}$

80. $a_n = (3^n + 5^n)^{1/n}$

81. $a_n = \tan^{-1} n$

82. $a_n = \frac{1}{\sqrt{n}}\tan^{-1} n$

83. $a_n = \left(\frac{1}{3}\right)^n + \frac{1}{\sqrt{2^n}}$

84. $a_n = \sqrt[n]{n^2 + n}$

85. $a_n = \frac{(\ln n)^{200}}{n}$

86. $a_n = \frac{(\ln n)^5}{\sqrt{n}}$

87. $a_n = n - \sqrt{n^2 - n}$

88. $a_n = \frac{1}{\sqrt{n^2-1} - \sqrt{n^2+n}}$

91-98 'de dizileri yakınsak sayarak limitlerini bulunuz:

91. $a_1 = 2, \quad a_{n+1} = \frac{72}{1 + a_n}$

92. $a_1 = -1, \quad a_{n+1} = \frac{a_n + 6}{a_n + 2}$

93. $a_1 = -4, \quad a_{n+1} = \sqrt{8 + 2a_n}$

94. $a_1 = 0, \quad a_{n+1} = \sqrt{8 + 2a_n}$

95. $a_1 = 5, \quad a_{n+1} = \sqrt{5a_n}$

96. $a_1 = 3, \quad a_{n+1} = 12 - \sqrt{a_n}$

97. $2, 2 + \frac{1}{2}, 2 + \frac{1}{2 + \frac{1}{2}}, 2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}, \ldots$

98. $\sqrt{1}, \sqrt{1 + \sqrt{1}}, \sqrt{1 + \sqrt{1 + \sqrt{1}}}, \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1}}}}, \ldots$

Genel terimi verilmiş dizinin ilk beş ardışık terimini yazınız:

5.213. $x_n = 1 + (-1)^n \dfrac{1}{n}$. **5.214.** $x_n = n(1 - (-1)^n)$.

5.215. $x_n = \dfrac{3n+5}{2n-3}$. **5.216.** $x_n = (-1)^n \arcsin \dfrac{\sqrt{3}}{2} + \pi n$.

Dizinin genel teriminin formülünü bulunuz:

5.217. $-\dfrac{1}{2}, \dfrac{1}{3}, -\dfrac{1}{4}, \dfrac{1}{5}, \ldots$ **5.218.** $0, 2, 0, 2, \ldots$

5.219. $2, \dfrac{4}{3}, \dfrac{6}{5}, \dfrac{8}{7}, \ldots$

5.220. $1, 0, -3, 0, 5, 0, -7, 0, \ldots$

5.221. $-3, \dfrac{5}{3}, -\dfrac{7}{5}, \dfrac{9}{7}, -\dfrac{11}{9}, \ldots$

5.222. $0, \dfrac{\sqrt{2}}{2}, 1, \dfrac{\sqrt{2}}{2}, 0, -\dfrac{\sqrt{2}}{2}, -1, -\dfrac{\sqrt{2}}{2}, 0, \ldots$

5.223–5.228 'de genel terimi verilmiş, üstten (alttan) sınırlı dizinin en büyük (en küçük) terimini bulunuz:

5.223. $x_n = 6n - n^2 - 5$. **5.224.** $x_n = e^{10n-n^2-24}$.

5.225. $x_n = \dfrac{\sqrt{n}}{9+n}$. **5.226.** $x_n = 3n^2 - 10n - 14$.

5.227. $x_n = 2n + \dfrac{512}{n^2}$. **5.228.** $x_n = -\dfrac{n^2}{2^n}$.

Limitleri bulunuz:

5.231. $\lim\limits_{n\to\infty} \dfrac{n-1}{3n}$. **5.232.** $\lim\limits_{n\to\infty} \dfrac{5n+1}{7-9n}$.

5.233. $\lim\limits_{n\to\infty} \dfrac{(n+1)^2}{2n^3}$. **5.234.** $\lim\limits_{n\to\infty} \dfrac{3n^2-7n+1}{2-5n-6n^2}$.

5.235. $\lim\limits_{n\to\infty} \dfrac{(n+2)^3-(n-2)^3}{95n^3+39n}$.

5.236. $\lim\limits_{n\to\infty} \left(\dfrac{2n-1}{5n+7} - \dfrac{1+2n^3}{2+5n^3}\right)$.

5.237. $\lim\limits_{n\to\infty} \dfrac{\sqrt[3]{n^4+3n+1}}{n-1}$. **5.238.** $\lim\limits_{n\to\infty} (\sqrt{n+2}-\sqrt{n})$.

5.239. $\lim\limits_{n\to\infty} n^{3/2}(\sqrt{n^3+1}-\sqrt{n^3-2})$. **5.240.** $\lim\limits_{n\to\infty} \dfrac{2^n+3^n}{2^n-3^n}$.

5.241. $\lim\limits_{n\to\infty} \left(\dfrac{1}{n^2}+\dfrac{2}{n^2}+\ldots+\dfrac{n-1}{n^2}\right)$.

5.242. $\lim\limits_{n\to\infty} \dfrac{1^2+2^2+\ldots+n^2}{n^3}$. **5.243.** $\lim\limits_{n\to\infty} \dfrac{\sqrt[3]{n^2}\sin(n^2)}{n-1}$.

5.244. $\lim\limits_{n\to\infty} \left(\dfrac{1}{1\cdot 2}+\dfrac{1}{2\cdot 3}+\ldots+\dfrac{1}{n(n+1)}\right)$.

5.245. $\lim\limits_{n\to\infty} \left(\dfrac{1}{5}-\dfrac{1}{25}+\ldots+(-1)^{n-1}\dfrac{1}{5^n}\right)$.

1) $\lim\limits_{n\to\infty} \left(\dfrac{1}{1\cdot 3}+\dfrac{1}{3\cdot 5}+\ldots+\dfrac{1}{(2n-1)(2n+1)}\right)$;

2) $\lim\limits_{n\to\infty} \dfrac{1}{\sqrt{n}}\left(\dfrac{1}{\sqrt{1}+\sqrt{3}}+\dfrac{1}{\sqrt{3}+\sqrt{5}}+\ldots+\dfrac{1}{\sqrt{2n-1}+\sqrt{2n+1}}\right)$;

3) $\lim\limits_{n\to\infty} \left(\dfrac{1}{1\cdot 2\cdot 3}+\dfrac{1}{2\cdot 3\cdot 4}+\ldots+\dfrac{1}{n(n+1)(n+2)}\right)$.

4) $\lim\limits_{n\to\infty} \left(\dfrac{2002}{n}\right)^n$; 2) $\lim\limits_{n\to\infty} \left(\dfrac{a}{n}\right)^n$ (a – reel sayıdır);

5) $\lim\limits_{n\to\infty} \left(\dfrac{2n+3}{n^2}\right)^n$; 4) $\lim\limits_{n\to\infty} \left(\dfrac{n+10}{2n-1}\right)^n$.

Genel terimi verilmiş dizinin tüm kısmi limitlerinini bulunuz:

5.251. $x_n = \dfrac{2+(-1)^n}{2-(-1)^n}$. **5.252.** $x_n = \cos\dfrac{\pi n}{4}$.

5.253. $x_n = \arcsin\dfrac{(-1)^n}{2}$.

1) $\dfrac{(-1)^n}{n+1}$; 2) $\dfrac{n^2}{n+5}$; 3) $\dfrac{1-n^3}{1+n^2}$; 4) $(-1)^n$; 5) $3^{(-1)^n n}$;
6) $\sin(\pi n/4)$; 7) $n\cos(\pi n/2)$.

Genel terimi verilmiş dizilerden hangilerinin sonsuz büyük dizi olduğunu belirleyiniz:

5.247. $x_n = 2^{\sqrt{n}}$. **5.248.** $x_n = n^{(-1)^n}$.

5.249. $x_n = n\sin\dfrac{\pi n}{2}$. **5.250.** $x_n = \lg(\lg n)$, $n \geqslant 2$.

$(x_n)_{n\in\mathbb{N}}$ dizilerinin her biri için $\inf\{x_n\}$, $\sup\{x_n\}$, $\underline{\lim\limits_{n\to\infty}}\, x_n$ ve $\overline{\lim\limits_{n\to\infty}}\, x_n$ 'i bulunuz:

5.255. $x_n = 1 + \dfrac{1}{n}$. **5.256.** $x_n = \dfrac{n+1}{n}\cos^2\dfrac{\pi n}{4}$.

5.257. $x_n = (-1)^n(2n+1)$.

5.258. $x_n = \dfrac{n+2}{n-2}\sin\dfrac{\pi n}{3}$, $n \geqslant 2$. **5.259.** $x_n = \dfrac{2+(-1)^n}{2} - \dfrac{1}{n}$.

Genel terimi verilmiş dizilerin sonsuz küçük dizi olduğunu gösteriniz:

1) $x_n = \dfrac{n^2-1}{n^3}$; 2) $x_n = \dfrac{2n+3}{n^2}$; 3) $x_n = \dfrac{q^n}{n}$, $|q| \leqslant 1$;
4) $x_n = \dfrac{2n+1}{(n+1)2^n}$; 5) $x_n = \dfrac{\sin n}{\sqrt{n}}$.

$\{x_n\}$ dizisinin ıraksak olduğunu gösteriniz:

1) $x_n = (-1)^n$; 2) $x_n = n$; 3) $x_n = \sin\dfrac{\pi n}{2}$;
4) $x_n = E\left(\dfrac{(-1)^n}{n}\right)$; 5) $x_n = \sin n$; 6) $x_n = \dfrac{n}{n+1}\cos\dfrac{2\pi n}{3}$.

Aşağıdaki dizilerin Cauchy dizisi olduğunu gösteriniz:

1) $x_n = \dfrac{1}{n}$, $n \in N$; 2) $x_n = \dfrac{n+1}{3n-2}$, $n \in N$;
3) $x_n = 0,\underbrace{77...7}_{n \text{ цифр}}$, $n \in N$;
4) $x_n = a + aq + ... + aq^{n-1}$, $|q| < 1$, $n \in N$; ise
5) $\{x_n\} = \left\{1, -1, \dfrac{1}{2}, -\dfrac{1}{2}, \dfrac{1}{3}, -\dfrac{1}{3}, ..., \dfrac{1}{n}, -\dfrac{1}{n}, ...\right\}$;
6) $x_1 = 1$, $x_n = x_{n-1} + \dfrac{(-1)^{n-1}}{n!}$ $(n = 2, 3, ...)$.

Cauchy kriterini kullanarak $\{x_n\}$ dizisinin ıraksak olduğunu gösteriniz:

1) $0{,}2^{(-1)^n n}$; 2) $\dfrac{n\cos\pi n - 1}{2n}$; 3) $(-1)^n\left(1 + \dfrac{1}{n}\right)^n$;
4) $\left(1 + \dfrac{(-1)^n}{n}\right)^n$; 5) $\dfrac{1}{2^2} + \dfrac{2}{3^2} + ... + \dfrac{n}{(n+1)^2}$.

Dizilerin monoton ve yakınsak olduğunu gösteriniz ve limitini bulunuz:

1) $x_1 = 4$, $x_{n+1} = \sqrt{6 + x_n}$; 2) $x_1 = 1$, $x_{n+1} = \sqrt[3]{6 + x_n}$;
3) $x_1 = 3$, $x_{n+1} = \sqrt{12 + x_n}$, $n \in N$.

Serilerin yakınsak olduğunu gösteriniz ve toplamını bulunuz:

12.1. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$.

12.2. $\sum_{n=2}^{\infty} \frac{2}{4n^2-9}$.

12.3. $\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$.

12.4. $\sum_{n=3}^{\infty} \frac{2n+1}{n(n^2-1)}$.

12.5*. $\sum_{n=1}^{\infty} \frac{\cos in}{3^n}$.

12.6. $\sum_{n=0}^{\infty} \frac{(1+i)^n}{2^n}$.

Yakınsaklığın gerekli şartını veya Cauchy kriterini kullanarak serilerin ıraksak olduğunu gösteriniz:

12.7. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}$.

12.8. $\sum_{n=1}^{\infty} \frac{n}{n^2+2}$.

12.9. $\sum_{n=1}^{\infty} (-1)^n$.

12.10. $\sum_{n=1}^{\infty} \frac{e^n}{n^{10}}$.

Karşılaştırma testini veya bu testin limit formunu kullanarak serilerin yakınsaklığını inceleyiniz:

12.19. $\sum_{n=1}^{\infty} \frac{1}{3n-2}$.

12.20. $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$.

12.21. $\sum_{n=1}^{\infty} \frac{n}{3n^3-1}$.

12.22. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)(n+2)}}$.

12.23. $\sum_{n=1}^{\infty} \sin\frac{\pi}{n}$.

12.24. $\sum_{n=1}^{\infty} \frac{n^2+3}{4n^3+5n}$.

12.25. $\sum_{n=3}^{\infty} n^2 \operatorname{tg}^6 \frac{\pi}{n}$.

12.26. $\sum_{n=1}^{\infty} \sqrt[3]{n^2} \operatorname{arctg} \frac{1}{n^2}$.

12.27. $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt[3]{n^2}}$.

12.28. $\sum_{n=1}^{\infty} e^{-n^2}$.

12.29. $\sum_{n=1}^{\infty} \frac{2^n}{(n+1)3^n}$.

12.30. $\sum_{n=1}^{\infty} \frac{\cos\sqrt{n} + i\sin\sqrt{n}}{n^2}$.

D'Alambert oran testine göre serinin yakınsaklığını inceleyiniz:

12.31. $\sum_{n=1}^{\infty} \frac{n^2+5}{2^n}$.

12.32. $\sum_{n=1}^{\infty} \frac{n^n}{n!}$.

12.33. $\frac{3}{1} + \frac{3\cdot 5}{1\cdot 4} + \ldots + \frac{3\cdot 5\ldots(2n+1)}{1\cdot 4\ldots(3n-2)} + \ldots$

12.34. $\sum_{n=1}^{\infty} \frac{n^5}{5^n}$.

12.35. $\sum_{n=1}^{\infty} \frac{(3n+1)!}{8^n n^2}$.

12.36. $\sum_{n=1}^{\infty} \frac{e^{2n+1}}{n!}$.

12.37. $\sum_{n=2}^{\infty} \frac{1 \cdot 3 \cdot 5 \ldots (2n-1)}{2^{2n}(n-1)!}$.

D' Alambert oran testini kullanarak serinin yakınsaklığını inceleyiniz:

12.31. $\sum_{n=1}^{\infty} \frac{n^2+5}{2^n}$.

12.32. $\sum_{n=1}^{\infty} \frac{n^n}{n!}$.

12.33. $\frac{3}{1} + \frac{3 \cdot 5}{1 \cdot 4} + \ldots + \frac{3 \cdot 5 \ldots (2n+1)}{1 \cdot 4 \ldots (3n-2)} + \ldots$

12.34. $\sum_{n=1}^{\infty} \frac{n^5}{5^n}$.

12.35. $\sum_{n=1}^{\infty} \frac{(3n+1)!}{8^n n^2}$.

12.36. $\sum_{n=1}^{\infty} \frac{e^{2n+1}}{n!}$.

12.37. $\sum_{n=2}^{\infty} \frac{1 \cdot 3 \cdot 5 \ldots (2n-1)}{2^{2n}(n-1)!}$.

12.38. $\sum_{n=1}^{\infty} \frac{\sin in}{3^n}$.

12.39. $\sum_{n=1}^{\infty} \frac{n^n}{n!(e-i)^n}$.

Cauchy oran testini kullanarak serinin yakınsaklığını inceleyiniz:

12.40. $\sum_{n=1}^{\infty} \left(\frac{n-1}{2n+1}\right)^n$.

12.41. $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{n^2}$.

12.42. $\sum_{n=1}^{\infty} \frac{1}{2^n} \left(1+\frac{1}{n}\right)^{n^2}$.

12.43. $\sum_{n=1}^{\infty} \frac{1}{3^n} \left(1+\frac{1}{n}\right)^{n^2}$.

12.44. $\sum_{n=1}^{\infty} \sqrt{n} \left(\frac{n}{4n-3}\right)^{2n}$.

12.45. $\sum_{n=1}^{\infty} n \left(\arcsin \frac{1}{n}\right)^n$.

12.46. $\sum_{n=1}^{\infty} u_n$, где $u_{2k-1} = \left(\frac{k}{2k+1}\right)^k$, $u_{2k} = \left(\frac{k}{3k-1}\right)^{k/2}$.

12.47. $\sum_{n=1}^{\infty} \left(\frac{n+2i}{(1+i)n+3}\right)^n$.

12.48. $\sum_{n=1}^{\infty} \left(\frac{i(2n+i)}{4n}\right)^n$.

Cauchy integral testini kullanarak serinin yakınsaklığını inceleyiniz:

12.49. $\sum_{n=2}^{\infty} \frac{1}{n \ln^2 n}$.

12.50. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$.

12.51. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$.

12.52. $\sum_{n=3}^{\infty} \frac{1}{n \ln n (\ln \ln n)^2}$.

Serilerin yakınsaklığını inceleyiniz:

12.53. $\sum_{n=1}^{\infty}\left(\frac{1+n}{1+n^2}\right)^2$.

12.54. $\sum_{n=2}^{\infty}\frac{1}{\sqrt{n}}\ln\frac{n+1}{n-1}$.

12.55. $\sum_{n=1}^{\infty}\frac{n^2}{3^n}$.

12.56. $\sum_{n=1}^{\infty}\frac{1}{n!}$.

12.57. $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^n$.

12.58. $\sum_{n=1}^{\infty}\frac{n}{n^3+1}$.

12.59. $\sum_{n=1}^{\infty}\frac{n^n}{(2n-1)^{n-1}}$.

12.60. $\sum_{n=1}^{\infty}\left(1-\frac{1}{n}\right)^{n^2}$.

12.61. $100+\frac{100\cdot 103}{1\cdot 5}+\ldots+\frac{100\cdot 103\ldots(97+3n)}{1\cdot 5\cdot 9\ldots(4n-3)}+\ldots$

12.62. $1+\frac{1\cdot 11}{3!}+\frac{1\cdot 11\cdot 21}{5!}+\ldots+\frac{1\cdot 11\cdot 21\ldots(10n-9)}{(2n-1)!}+\ldots$

12.63. $\frac{1}{2}+\frac{1\cdot 5}{2\cdot 4\cdot 6}+\frac{1\cdot 5\cdot 9}{2\cdot 4\cdot 6\cdot 8\cdot 10}+\ldots+\frac{1\cdot 5\cdot 9\ldots(4n-3)}{(4n-2)!!}+\ldots$

12.64. $\sum_{n=2}^{\infty}\frac{\sqrt{n+1}-\sqrt{n-1}}{n^{3/4}}$.

12.65. $\sum_{n=2}^{\infty}\left(\frac{n-1}{n+1}\right)^{n(n-1)}$.

12.66. $\sum_{n=1}^{\infty}\left(\ln\frac{1}{n^{2/5}}-\ln\left(\sin\frac{1}{n^{2/5}}\right)\right)$.

12.67. $\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}$.

12.68. $\sum_{n=1}^{\infty}\sin\frac{\pi}{2n^2}$.

12.69. $\sum_{n=1}^{\infty}\ln\left(1+\frac{1}{n^2}\right)$.

12.70. $\sum_{n=3}^{\infty}\frac{1}{n\ln n\ln\ln n}$.

12.71. $\sum_{n=1}^{\infty}\frac{3^n\cdot n!}{n^n}$.

12.72*. $\sum_{n=1}^{\infty}\frac{e^n\cdot n!}{n^n}$.

12.73. $2+\frac{2\cdot 5}{1\cdot 5}+\frac{2\cdot 5\cdot 8}{1\cdot 5\cdot 9}+\ldots+\frac{2\cdot 5\cdot 8\ldots(3n-1)}{1\cdot 5\cdot 9\ldots(4n-3)}+\ldots$

12.74. $\sum_{n=1}^{\infty}\frac{n^{100}}{2^n}$.

12.75. $\sum_{n=1}^{\infty}\ln\left(1+\frac{1}{n}\right)$.

12.76. $\sum_{n=1}^{\infty}\left(\frac{2n-1}{5n+3}\right)^n$.

12.77. $\sum_{n=1}^{\infty}\left(\frac{n}{n+2}\right)^n$.

12.78. $\frac{1}{100}+\frac{1\cdot 4}{100\cdot 102}+\frac{1\cdot 4\cdot 7}{100\cdot 102\cdot 104}+\ldots+\frac{1\cdot 4\cdot 7\ldots(3n-2)}{100\cdot 102\ldots(98+2n)}+\ldots$

Raabe veya Gauss testi yardımıyla genel terimi verilmiş serinin yakınsaklığını inceleyiniz:

1) $a_n = \frac{(2n-1)!!}{(2n)!!}$; 2) $a_n = \left(\frac{(2n+1)!!}{(2n+2)!!}\right)^{\alpha} \frac{1}{n^{\beta}}$;

3) $a_n = \left(\frac{(a+1)(a+2)...(a+n)}{(b+1)(b+2)...(b+n)}\right)^{\alpha}$, $a>0$, $b>0$; 4) $a_n = \frac{n!\,e^n}{n^{n+\alpha}}$;

5) $a_n = \frac{\sqrt{n!}}{(a+\sqrt{2})(a+\sqrt{3})...(a+\sqrt{n+1})}$, $a>0$;

6) $a_n = \frac{(n+1)!}{\beta(\beta+1)...(\beta+n)n^{\alpha}}$, $\beta>0$;

7) $a_n = \frac{1\cdot 4...(3n-2)\cdot 2\cdot 5...(3n+2)}{n!(n+1)!9^n}$;

8) $a_n = \frac{\ln 2 \ln 3... \ln(n+1)}{\ln(2+a)\ln(3+a)... \ln(n+1+a)}$, $a>0$.

Logaritmik test yarımıyla genel terimi verilmiş serinin yakınsaklığını inceleyiniz:

4) $a_n = \frac{1}{(\ln n)^{\ln n}}$, $n \geqslant 2$; 5) $a_n = \frac{1}{(\ln\ln n)^{\ln n}}$, $n \geqslant 3$;

6) $a_n = \frac{1}{(\ln n)^{\ln\ln n}}$, $n \geqslant 3$; 7) $a_n = \frac{n^{\ln n}}{(\ln n)^n}$, $n \geqslant 2$;

Raabe veya Gauss testini kullanarak genel terimi verilmiş $\sum_{n=1}^{\infty} a_n$ serisinin yakınsaklığını inceleyiniz:

1) $a_n = \frac{(2n-1)!!}{(2n)!!}$; 2) $a_n = \left(\frac{(2n+1)!!}{(2n+2)!!}\right)^{\alpha} \frac{1}{n^{\beta}}$;

3) $a_n = \left(\frac{(a+1)(a+2)...(a+n)}{(b+1)(b+2)...(b+n)}\right)^{\alpha}$, $a>0$, $b>0$; 4) $a_n = \frac{n!e^n}{n^{n+\alpha}}$;

5) $a_n = \frac{\sqrt{n!}}{(a+\sqrt{2})(a+\sqrt{3})...(a+\sqrt{n+1})}$, $a>0$; 6) $a_n = \frac{(n+1)!}{\beta(\beta+1)...(\beta+n)n^{\alpha}}$, $\beta>0$;

7) $a_n = \frac{1\cdot 4\cdot ...\cdot (3n-2)\cdot 2\cdot 5\cdot ...\cdot (3n+2)}{n!(n+1)!9^n}$; 8) $a_n = \frac{\ln 2\cdot \ln 3\cdot ...\cdot \ln(n+1)}{\ln(2+a)\ln(3+a)\cdot ...\cdot \ln(n+a+1)}$, $a>0$;

Bertrand Logaritmik testine göre genel terimi verilmiş serinin yakınsaklığını inceleyiniz:

1) $a_n = \frac{1}{(\ln n)^{\ln n}}$, $n \geq 2$; 2) $a_n = \frac{1}{(\ln\ln n)^{\ln n}}$, $n \geq 3$;

3) $a_n = \frac{1}{(\ln n)^{\ln\ln n}}$, $n \geq 3$; 4) $a_n = \frac{n^{\ln n}}{(\ln n)^n}$, $n \geq 2$;

A.L. Cauchy benzerlik testine göre serinin yakınsaklığını inceleyiniz:

1) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\ln^2 2n}$; 2) $\sum_{n=3}^{\infty} \frac{\ln n}{n^2 \ln\ln n}$; 3) $\sum_{n=2}^{\infty} \frac{\sqrt{1+\ln n}}{n(1+\ln^2 n)}$;)

Serilerin mutlak veya şartlı yakınsaklığını inceleyiniz:

12.90. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{3n-1}$. **12.91.** $\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$.

12.92. $\sum_{n=1}^{\infty}(-1)^n\frac{n}{5n-2}$. **12.93.** $\sum_{n=1}^{\infty}(-1)^n\left(\frac{2n-1}{3n+2}\right)^n$.

12.94. $\frac{1}{3}-\frac{1\cdot4}{3\cdot5}+\frac{1\cdot4\cdot7}{3\cdot5\cdot7}-\ldots+(-1)^{n+1}\frac{1\cdot4\cdot7\ldots(3n-2)}{3\cdot5\cdot7\ldots(2n+1)}+\ldots$

12.95. $\sum_{n=2}^{\infty}(-1)^n\frac{\ln n}{n}$. **12.96.** $\sum_{n=1}^{\infty}(-1)^n\frac{n!}{1\cdot3\cdot5\ldots(2n-1)}$.

12.97. $\sum_{n=3}^{\infty}(-1)^n\frac{1}{n\ln n(\ln\ln n)^3}$. **12.98.** $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^\alpha}$.

12.99. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n^2}{2^n}$. **12.100.** $\sum_{n=2}^{\infty}(-1)^n\frac{1}{n-\ln n}$.

12.101. $\sum_{n=1}^{\infty}\frac{\cos n\alpha}{n^2}$. **12.102.** $\sum_{n=1}^{\infty}(-1)^n\frac{2^{n^2}}{n!}$.

12.103. $\sum_{n=3}^{\infty}(-1)^n\frac{1}{n\ln n\sqrt{\ln\ln n}}$.

12.104. $\sum_{n=1}^{\infty}\frac{\sin n\alpha}{(\ln3)^n}$. **12.105*.** $\sum_{n=1}^{\infty}\frac{\sin\frac{\pi n}{4}}{n}$.

1. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3n+1}{n(n+2)}$.
2. $\sum_{n=1}^{\infty}(-1)^n\frac{2n^2}{n^5+n^2+1}$.
3. $\sum_{n=1}^{\infty}(-1)^n\frac{n^2+1}{\sqrt{n^5+3n^2+2}}$.
4. $\sum_{n=1}^{\infty}(-1)^n\frac{\sin(n^2\sqrt{n})}{n^2\sqrt{n}}$.
5. $\sum_{n=1}^{\infty}(-1)^n\frac{2n^2-1}{3n^3}$.
6. $\sum_{n=1}^{\infty}(-1)^n\frac{n^2+3}{\sqrt{n^5}}$.
7. $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{n}{3n+1}\right)^{n^2}$.
8. $\sum_{n=1}^{\infty}(-1)^n\left(\frac{3n+2}{2n}\right)^n$.
9. $\sum_{n=1}^{\infty}(-1)^n\sin^2\frac{\pi}{3^n}$.
10. $\sum_{n=1}^{\infty}\frac{\cos\pi n}{\sqrt{n+2}}$.

Fonksiyonel dizinin E kümesinde noktasal yakınsak olduğunu gösteriniz ve f(x) limitini bulunuz:

1. 1) $f_n(x)=x^n-3x^{n+2}+2x^{n+3}$, $E=[0;1]$;
2) $f_n(x)=x^4\cos\frac{1}{nx}$, $E=(0;+\infty)$;
3) $f_n(x)=\frac{nx^2}{x+3n+2}$, $E=[0;+\infty)$;
4) $f_n(x)=\sqrt{x^2+1/\sqrt{n}}$, $E=R$;
5) $f_n(x)=(x-1)\operatorname{arctg}x^n$, $E=(0;+\infty)$;
6) $f_n(x)=\sqrt[n]{1+x^n}$, $E=[0;2]$.

2. 1) $f_n(x) = n^3x^2e^{-nx}$, $E = [0;+\infty)$;
2) $f_n(x) = n(\sqrt{x^2+1/n} - x)$, $E = (0;+\infty)$;
3) $f_n(x) = n(x^{1/n} - 1)$, $E = [1;3]$;
4) $f_n(x) = n\operatorname{arcctg} nx^2$, $E = (0;+\infty)$;
5) $f_n(x) = n(x^{1/n} - x^{1/2n})$, $E = (0;+\infty)$;
6) $f_n(x) = \sqrt[n]{1+x^n+(x^2/2)^n}$, $E = [0;+\infty)$.

Fonksiyonel dizinin E kümesinde düzgün yakınsak olduğunu gösteriniz:

3. 1) $f_n(x) = e^{-nx^2}$, $E = [1;+\infty)$;
2) $f_n(x) = x^{2n}$, $E = [0;\delta]$, $0 < \delta < 1$;
3) $f_n(x) = \sqrt{n}\sin\dfrac{x}{n\sqrt{n}}$, $E = R$;
4) $f_n(x) = \dfrac{\sin n\sqrt{x}}{\ln(n+1)}$, $E = [0;+\infty)$;
5) $f_n(x) = \dfrac{n}{x^2+n^2}\operatorname{arctg}\sqrt{nx}$, $E = [0;+\infty)$;
6) $f_n(x) = \ln\left(1 + \dfrac{\cos nx}{\sqrt{n+x}}\right)$, $E = [0;+\infty)$.

4. 1) $f_n(x) = \dfrac{nx^2}{n+x}$, $E = [1;+\infty)$;
2) $f_n(x) = \sqrt{x^2+1/n}$, $E = R$;
3) $f_n(x) = e^{-(x-n)^2}$, $E = [-1;1]$;
4) $f_n(x) = \dfrac{nx}{1+n^3x^2}$, $E = [1;+\infty)$;
5) $f_n(x) = n^{3/4}xe^{-\sqrt{nx}}$, $E = [0;+\infty)$;
6) $f_n(x) = \dfrac{n^2x^2}{1+n^2x^4}\sin\dfrac{x^2}{\sqrt{n}}$, $E = [1;+\infty)$.

Serinin noktasal yakınsaklık ve mutlak yakınsaklık bölgelerini bulunuz:

1. 1) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{x^n}$; 2) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{\pi x}{n}$; 3) $\displaystyle\sum_{n=1}^{\infty}e^{-nx}$;
4) $\displaystyle\sum_{n=1}^{\infty}\frac{\cos\pi nx}{n\ln^2(n+1)}$; 5) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n(x+2)^n}$; 6) $\displaystyle\sum_{n=1}^{\infty}\frac{\ln^n x}{n^2}$.

2. 1) $\displaystyle\sum_{n=1}^{\infty}(5-x^2)^n$; 2) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^x}$; 3) $\displaystyle\sum_{n=1}^{\infty}n^{-\ln x^2}$; 4) $\displaystyle\sum_{n=1}^{\infty}n^2e^{nx^2}$;
5) $\displaystyle\sum_{n=1}^{\infty}\frac{n}{\ln^n(x+2)}$; 6) $\displaystyle\sum_{n=1}^{\infty}\frac{\operatorname{tg}^n x}{n^2+4}$.

3. 1) $\displaystyle\sum_{n=1}^{\infty}\frac{n}{n+1}\ln^n(x^2+2)$; 2) $\displaystyle\sum_{n=1}^{\infty}\frac{n}{n^2+4}\left(\frac{x+2}{2x+1}\right)^n$;
3) $\displaystyle\sum_{n=1}^{\infty}\sqrt{\frac{n+2}{n}}\left(\frac{x^2-5x+6}{x^2+5x+6}\right)^n$; 4) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\left(\frac{2x}{x^2+1}\right)^n$;
5) $\displaystyle\sum_{n=1}^{\infty}n^2\left(\frac{2x-3}{4}\right)^n$; 6) $\displaystyle\sum_{n=1}^{\infty}\frac{2^n\sin^n x}{n(n+1)}$.

4. 1) $\displaystyle\sum_{n=1}^{\infty}\frac{\cos nx}{\sqrt[3]{n}}$; 2) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{x^2+\sqrt{n}}$; 3) $\displaystyle\sum_{n=1}^{\infty}e^{-nx}\sin nx$;
4) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}e^{-n\sin x}$; 5) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n}{3}\ln\left(1+\frac{x}{n}\right)\right)^n$;
6) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n}{2}\right)^n(e^{x/n}-1)^n$.

5. 1) $\displaystyle\sum_{n=1}^{\infty}\sin\pi nx$; 2) $\displaystyle\sum_{n=1}^{\infty}\left(1+\frac{x}{n}\right)^n n^{-x}$; 3) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{x^{2n}+1}$;
4) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{x^2}{n}+x\right)^n$; 5) $\displaystyle\sum_{n=1}^{\infty}\left(\cos\frac{\pi x}{n}\right)^{n^3}$; 6) $\displaystyle\sum_{n=1}^{\infty}\frac{x^n}{1-x^n}$.

6. 1) $\sum_{n=1}^{\infty} \sin(\pi\sqrt{n^2+x^2})$; 2) $\sum_{n=1}^{\infty} \frac{\sin n \sin n^2}{n+\sqrt{x}}$; 3) $\sum_{n=1}^{\infty} \underbrace{\sin \sin \ldots \sin}_{n \text{ раз}} x$;

4) $\sum_{n=1}^{\infty} \frac{x(x-1)\ldots(x-(n-1))}{n!}$; 5) $\sum_{n=1}^{\infty} \frac{n!}{(x^2+1)(x^2+1)\ldots(x^2+n)}$;

6) $\sum_{n=1}^{\infty} \frac{x^n}{(1+x)(1+x^2)\ldots(1+x^n)}$.

Fonksiyon serilerinin noktasal yakınsaklık bölgelerini bulunuz:

1. $\sum_{n=1}^{\infty} \frac{n}{(\sqrt[3]{n}+1)^{x+3}}$.
2. $\sum_{n=1}^{\infty} \frac{2^n}{n(x^2-6x+10)^n}$.
3. $\sum_{n=1}^{\infty} \frac{n^4}{n^{x^2+4x}+3}$.
4. $\sum_{n=1}^{\infty} \frac{3^n}{n(x^2-5x+9)^n}$.
5. $\sum_{n=1}^{\infty} \frac{n}{n^{x^2-2}+1}$.
6. $\sum_{n=1}^{\infty} \frac{4^n}{n^2(x^2+3)^n}$.
7. $\sum_{n=1}^{\infty} \frac{n^2}{(n+\sqrt{n})^x}$.
8. $\sum_{n=1}^{\infty} \frac{5^n}{n(x^2-4x+8)^n}$.
9. $\sum_{n=1}^{\infty} \frac{n^2+1}{n^{3+3x-x^2}}$.
10. $\sum_{n=1}^{\infty} \frac{6^n}{n^2(x^2-2x+6)^n}$.

Weierstrass testine göre serinin gösterilen aralıkta düzgün yakınsal olduğunu gösteriniz:

8. 1) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$, $-1 \leqslant x \leqslant 1$; 2) $\sum_{n=1}^{\infty} \frac{1}{(n+x)^2}$, $0 \leqslant x < +\infty$;

3) $\sum_{n=1}^{\infty} \frac{x^2}{1+n^{3/2}x^2}$, $-\infty < x < +\infty$;

4) $\sum_{n=1}^{\infty} \frac{\sin^2 2nx}{\sqrt[3]{n^4+x^2}}$, $-\infty < x < +\infty$;

5) $\sum_{n=1}^{\infty} \frac{\operatorname{arctg} nx}{x^4+n\sqrt[3]{n}}$, $-\infty < x < +\infty$;

6) $\sum_{n=1}^{\infty} 2^{-n} \cos \pi n x$, $-\infty < x < +\infty$.

9. 1) $\sum_{n=1}^{\infty} e^{-\sqrt{nx}}$, $1 \leqslant x < +\infty$; 2) $\sum_{n=1}^{\infty} \frac{(3x)^n}{n\sqrt{n+x}}$, $0 \leqslant x \leqslant \frac{1}{3}$;

3) $\sum_{n=1}^{\infty} \frac{(x-1)^n}{(3n+1)3^n}$, $-1 \leqslant x \leqslant 3$;

4) $\sum_{n=1}^{\infty} \frac{(x+2)^n \cos^2 nx}{\sqrt{n^3+x^4}}$, $-3 \leqslant x \leqslant -1$;

5) $\sum_{n=1}^{\infty} \frac{(n+2)^3(2x)^{2n}}{x^2+3n+4}$, $-\frac{1}{4} \leqslant x \leqslant \frac{1}{4}$;

6) $\sum_{n=1}^{\infty} \frac{n \operatorname{arctg} 2n^2 x}{\sqrt[3]{n^7+n+x}}$, $0 \leqslant x < +\infty$.

10. 1) $\sum\limits_{n=1}^{\infty} \frac{\cos nx \sin(1/(nx))}{4+\ln^2 nx}$, $2 \leqslant x < +\infty$;

2) $\sum\limits_{n=1}^{\infty} n^3 e^{-n^2 x}$, $\delta < x < +\infty$, $\delta > 0$;

3) $\sum\limits_{n=1}^{\infty} \frac{R^n x^n}{1+x^{2n}}$, $R > 0$, $-\alpha < x < \alpha$, $\alpha < \min\left(1; \frac{1}{R}\right)$;

4) $\sum\limits_{n=1}^{\infty} n^{-x} \ln^2 n$, $\delta < x < +\infty$, $\delta > 1$;

5) $\sum\limits_{n=1}^{\infty} \frac{x}{4+n^3 x^2}$, $0 \leqslant x < +\infty$; 6) $\sum\limits_{n=1}^{\infty} \frac{nx}{1+n^5 x^2}$, $-\infty < x < +\infty$

Kuvvet serisinin yakınsaklık yarıçapını ve aralığını bulunuz:

1. $\sum\limits_{n=1}^{\infty} \frac{(x-2)^n}{n9^n}$.
2. $\sum\limits_{n=1}^{\infty} \frac{(x-3)^{2n+1}}{\sqrt[3]{n}}$.
3. $\sum\limits_{n=1}^{\infty} \frac{(x+5)^{2n}}{4^n}$.
4. $\sum\limits_{n=1}^{\infty} \frac{(x+1)^n}{(n+1)2^n}$.
5. $\sum\limits_{n=1}^{\infty} \frac{(x-2)^n}{(n+1)3^n}$.
6. $\sum\limits_{n=1}^{\infty} \frac{(x-3)^{2n-1}}{(2n^3+3n)4^n}$.
7. $\sum\limits_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}(x+3)^n$.
8. $\sum\limits_{n=1}^{\infty} \frac{n}{(n+1)^2 3^n}(x+2)^n$.
9. $\sum\limits_{n=1}^{\infty} \frac{(-1)^n}{(3n+2)2^n}(x+4)^n$
10. $\sum\limits_{n=1}^{\infty} \frac{n^3}{(n+1)!}(x+2)^{2n+1}$.

Kuvvet serisinin yakınsalık bölgesini ve bu bölgede toplamını bulunuz:

1. $\sum\limits_{n=1}^{\infty} \frac{6^n x^n}{n}$.
2. $\sum\limits_{n=1}^{\infty} \frac{x^{n+2}}{n(n+1)}$.
3. $\sum\limits_{n=0}^{\infty} \frac{(1-16x^4)^n}{n+1}$.
4. $\sum\limits_{n=1}^{\infty} \frac{3^{n-1}}{nx^{n-1}}$.
5. $\sum\limits_{n=0}^{\infty} \frac{2^n}{(n+1)x^{2n}}$.
6. $\sum\limits_{n=1}^{\infty} \frac{2^n x^{n+1}}{n(n+1)}$.
7. $\sum\limits_{n=1}^{\infty} \frac{3^n}{n(n+1)x^n}$.
8. $\sum\limits_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$.
9. $\sum\limits_{n=1}^{\infty} \frac{(-1)^n 4^n x^{2n}}{2n+1}$.
10. $\sum\limits_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n}}{n(2n-1)}$.
11. $\sum\limits_{n=1}^{\infty} nx^{n+1}$.
12. $\sum\limits_{n=1}^{\infty} n2^n x^n$.

13. $\sum_{n=0}^{\infty}(n+1)x^{2n+4}$.

14. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}nx^{n-1}}{3^{n-1}}$.

15. $\sum_{n=0}^{\infty}\frac{(n+1)(n+2)x^n}{2^n}$.

16. $\sum_{n=1}^{\infty}n^2x^{n-1}$.

17. $\sum_{n=0}^{\infty}(2n^2-2n+1)x^n$.

18. $\sum_{n=1}^{\infty}(-1)^{n-1}n(n+1)x^n$.

19. $\sum_{n=1}^{\infty}n(n+2)x^n$.

20. $\sum_{n=1}^{\infty}n(x^3+1)^{n-1}$.

Serinin yakınsaklık aralığını bulunuz, aralığın uç noktalarında mutlak ve şartlı yakınsaklığını bulunuz:

7. 1) $\sum_{n=1}^{\infty}\frac{(x-1)^n}{n\sqrt{n}}$; 2) $\sum_{n=1}^{\infty}\left(\frac{2n-1}{3n+2}\right)^n(x+2)^n$; 3) $\sum_{n=0}^{\infty}\frac{(-1)^nx^n}{2n+1}$;

4) $\sum_{n=0}^{\infty}\frac{(-1)^{n-1}2n+3}{3n^2+4}x^{2n+1}$; 5) $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}}\left(\frac{x-1}{3}\right)^n$;

6) $\sum_{n=1}^{\infty}\sqrt{\frac{n^4+3}{n^3+4n}}(x+2)^n$.

8. 1) $\sum_{n=1}^{\infty}\frac{5^n+(-3)^n}{n+1}x^n$; 2) $\sum_{n=1}^{\infty}\frac{(x+1)^n}{\sqrt{n+1}}\ln\frac{3n-2}{3n+2}$;

3) $\sum_{n=1}^{\infty}\frac{\sqrt[3]{2n+1}-\sqrt[3]{2n-1}}{\sqrt{n}}(x+3)^n$; 4) $\sum_{n=1}^{\infty}(\sqrt[n]{a}-1)x^n,\ a>0$;

5) $\sum_{n=0}^{\infty}3^n(n^3+2)(x-1)^{2n}$; 6) $\sum_{n=1}^{\infty}4^{n^2}(x+1)^{n^2}$.

9. 1) $\sum_{n=1}^{\infty}\frac{(2n-1)!!}{n!}(x+2)^n$; 2) $\sum_{n=1}^{\infty}\frac{(x+3)^{n^2}}{n^n}$;

3) $\sum_{n=1}^{\infty}\left(1-\frac{1}{n}\right)^{n^2}x^n$; 4) $\sum_{n=1}^{\infty}\left(1+\frac{1}{2}+\ldots+\frac{1}{n}\right)(x-1)^n$;

5) $\sum_{n=1}^{\infty}\frac{(2+(-1)^n)^n}{n}(x+1)^n$; 6) $\sum_{n=1}^{\infty}\frac{(1+2\cos(\pi n/4))^nx^n}{\ln^2(n+1)}$.

10. 1) $\sum_{n=1}^{\infty}\frac{(x-1)^n}{n^\alpha}$; 2) $\sum_{n=1}^{\infty}\left(\frac{(2n-1)!!}{(2n)!!}\right)^\alpha x^n$;

Fonksiyonların Maclaurin serisine açılımlarını bulunuz:

1. $\frac{5}{6-x-x^2}$.

2. $\ln(1-x-20x^2)$.

3. $\sin(x+\frac{\pi}{6})$.

4. $\frac{x}{\sqrt{9+x^2}}$.

5. $\frac{x}{3+2x}$.

6. $\sqrt[4]{16+x}$.

7. xe^{2x^2+1}.

8. $\frac{1}{\sqrt[3]{27-x}}$.

9. $\ln(12x^2+7x+1)$.

10. $\cos^2 3x$.

4. 1) e^{-x^2}; 2) $\dfrac{x^2}{(1+x)^2}$; 3) $\dfrac{1}{(1-x^3)^2}$; 4) $(1-x^2)^{-3/2}$;
5) $\sin\dfrac{x^2}{3}$; 6) $\operatorname{ch}(x\sqrt{2})$; 7) $\dfrac{x^3}{\sqrt{1-2x}}$; 8) $\sqrt{1-x^2}$.

5. 1) $e^{2x}+2e^{-x}$; 2) $(1+x)e^{-x}$; 3) $x^2\ln(4+x^2)$;
4) $(1-x)\ln(1-x)$; 5) $\ln\sqrt[3]{\dfrac{1+x}{1-x}}$; 6) $(1+x^2)\operatorname{arctg} x$;
7) $\dfrac{1}{2}(\operatorname{sh} x+\sin x)$; 8) $\arccos x$.