

Genel terimi verilmiş dizinin ilk beş ardışık terimini yazınız:

$$5.213. x_n = 1 + (-1)^n \frac{1}{n}. \quad 5.214. x_n = n(1 - (-1)^n).$$

$$5.215. x_n = \frac{3n+5}{2n-3}. \quad 5.216. x_n = (-1)^n \arcsin \frac{\sqrt{3}}{2} + \pi n.$$

Dizinin genel teriminin formülünü bulunuz:

$$5.217. -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, \dots \quad 5.218. 0, 2, 0, 2, \dots$$

$$5.219. 2, \frac{4}{3}, \frac{6}{5}, \frac{8}{7}, \dots$$

$$5.220. 1, 0, -3, 0, 5, 0, -7, 0, \dots$$

$$5.221. -3, \frac{5}{3}, -\frac{7}{5}, \frac{9}{7}, -\frac{11}{9}, \dots$$

$$5.222. 0, \frac{\sqrt{2}}{2}, 1, \frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2}, -1, -\frac{\sqrt{2}}{2}, 0, \dots$$

5.223–5.228 'de genel terimi verilmiş, üstten (alttan) sınırlı dizinin en büyük (en küçük) terimini bulunuz:

$$5.223. x_n = 6n - n^2 - 5. \quad 5.224. x_n = e^{10n - n^2 - 24}.$$

$$5.225. x_n = \frac{\sqrt{n}}{9+n}. \quad 5.226. x_n = 3n^2 - 10n - 14.$$

$$5.227. x_n = 2n + \frac{512}{n^2}. \quad 5.228. x_n = -\frac{n^2}{2^n}.$$

Limitleri bulunuz:

$$5.231. \lim_{n \rightarrow \infty} \frac{n-1}{3n}. \quad 5.232. \lim_{n \rightarrow \infty} \frac{5n+1}{7-9n}.$$

$$5.233. \lim_{n \rightarrow \infty} \frac{(n+1)^2}{2n^3}. \quad 5.234. \lim_{n \rightarrow \infty} \frac{3n^2 - 7n + 1}{2 - 5n - 6n^2}.$$

$$5.235. \lim_{n \rightarrow \infty} \frac{(n+2)^3 - (n-2)^3}{95n^3 + 39n}.$$

$$5.236. \lim_{n \rightarrow \infty} \left(\frac{2n-1}{5n+7} - \frac{1+2n^3}{2+5n^3} \right).$$

$$5.237. \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4 + 3n + 1}}{n - 1}. \quad 5.238. \lim_{n \rightarrow \infty} (\sqrt{n + 2} - \sqrt{n}).$$

$$5.239. \lim_{n \rightarrow \infty} n^{3/2}(\sqrt{n^3 + 1} - \sqrt{n^3 - 2}). \quad 5.240. \lim_{n \rightarrow \infty} \frac{2^n + 3^n}{2^n - 3^n}.$$

$$5.241. \lim_{n \rightarrow \infty} \left(\frac{1}{n^2} + \frac{2}{n^2} + \dots + \frac{n-1}{n^2} \right).$$

$$5.242. \lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + \dots + n^2}{n^3}. \quad 5.243. \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2} \sin(n^2)}{n - 1}.$$

$$5.244. \lim_{n \rightarrow \infty} \left(\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{n(n+1)} \right).$$

$$5.245. \lim_{n \rightarrow \infty} \left(\frac{1}{5} - \frac{1}{25} + \dots + (-1)^{n-1} \frac{1}{5^n} \right).$$

$$1) \lim_{n \rightarrow \infty} \left(\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \dots + \frac{1}{(2n-1)(2n+1)} \right);$$

$$2) \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} \left(\frac{1}{\sqrt{1} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{5}} + \dots + \frac{1}{\sqrt{2n-1} + \sqrt{2n+1}} \right);$$

$$3) \lim_{n \rightarrow \infty} \left(\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \dots + \frac{1}{n(n+1)(n+2)} \right).$$

$$4) \lim_{n \rightarrow \infty} \left(\frac{2002}{n} \right)^n; \quad 2) \lim_{n \rightarrow \infty} \left(\frac{a}{n} \right)^n \quad (a - \text{reel sayıdır});$$

$$5) \lim_{n \rightarrow \infty} \left(\frac{2n+3}{n^2} \right)^n; \quad 4) \lim_{n \rightarrow \infty} \left(\frac{n+10}{2n-1} \right)^n.$$

Genel terimi verilmiş dizinin tüm kısmi limitlerini bulunuz:

$$5.251. x_n = \frac{2 + (-1)^n}{2 - (-1)^n}. \quad 5.252. x_n = \cos \frac{\pi n}{4}.$$

$$5.253. x_n = \arcsin \frac{(-1)^n}{2}.$$

$$1) \frac{(-1)^n}{n+1}; \quad 2) \frac{n^2}{n+5}; \quad 3) \frac{1-n^3}{1+n^2}; \quad 4) (-1)^n; \quad 5) 3^{(-1)^n n};$$

$$6) \sin(\pi n/4); \quad 7) n \cos(\pi n/2).$$

Genel terimi verilmiş dizilerden hangilerinin sonsuz büyük dizi olduğunu belirleyiniz:

$$5.247. x_n = 2^{\sqrt{n}}. \quad 5.248. x_n = n^{(-1)^n}.$$

$$5.249. x_n = n \sin \frac{\pi n}{2}. \quad 5.250. x_n = \lg(\lg n), \quad n \geq 2.$$

$(x_n)_{n \in \mathbb{N}}$ dizilerinin her biri için $\inf \{x_n\}$, $\sup \{x_n\}$, $\lim_{n \rightarrow \infty} x_n$ ve $\overline{\lim}_{n \rightarrow \infty} x_n$ 'i bulunuz:

5.255. $x_n = 1 + \frac{1}{n}$. **5.256.** $x_n = \frac{n+1}{n} \cos^2 \frac{\pi n}{4}$.

5.257. $x_n = (-1)^n (2n+1)$.

5.258. $x_n = \frac{n+2}{n-2} \sin \frac{\pi n}{3}$, $n \geq 2$. **5.259.** $x_n = \frac{2+(-1)^n}{2} - \frac{1}{n}$.

Genel terimi verilmiş dizilerin sonsuz küçük dizi olduğunu gösteriniz:

1) $x_n = \frac{n^2-1}{n^3}$; 2) $x_n = \frac{2n+3}{n^2}$; 3) $x_n = \frac{q^n}{n}$, $|q| \leq 1$;
4) $x_n = \frac{2n+1}{(n+1)2^n}$; 5) $x_n = \frac{\sin n}{\sqrt{n}}$.

$\{x_n\}$ dizisinin ıraksak olduğunu gösteriniz:

1) $x_n = (-1)^n$; 2) $x_n = n$; 3) $x_n = \sin \frac{\pi n}{2}$;
4) $x_n = E\left(\frac{(-1)^n}{n}\right)$; 5) $x_n = \sin n$; 6) $x_n = \frac{n}{n+1} \cos \frac{2\pi n}{3}$.

Aşağıdaki dizilerin Cauchy dizisi olduğunu gösteriniz:

1) $x_n = \frac{1}{n}$, $n \in \mathbb{N}$; 2) $x_n = \frac{n+1}{3n-2}$, $n \in \mathbb{N}$;
3) $x_n = 0, \underbrace{77\dots7}_{n \text{ цифр}}$, $n \in \mathbb{N}$;
4) $x_n = a + aq + \dots + aq^{n-1}$, $|q| < 1$, $n \in \mathbb{N}$; ise
5) $\{x_n\} = \left\{1, -1, \frac{1}{2}, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{3}, \dots, \frac{1}{n}, -\frac{1}{n}, \dots\right\}$;
6) $x_1 = 1$, $x_n = x_{n-1} + \frac{(-1)^{n-1}}{n!}$ ($n = 2, 3, \dots$).

Cauchy kriterini kullanarak $\{x_n\}$ dizisinin ıraksak olduğunu gösteriniz:

1) $0, 2^{(-1)^n n}$; 2) $\frac{n \cos \pi n - 1}{2n}$; 3) $(-1)^n \left(1 + \frac{1}{n}\right)^n$;
4) $\left(1 + \frac{(-1)^n}{n}\right)^n$; 5) $\frac{1}{2^2} + \frac{2}{3^2} + \dots + \frac{n}{(n+1)^2}$.

Dizilerin monoton ve yakınsak olduğunu gösteriniz ve limitini bulunuz:

1) $x_1 = 4$, $x_{n+1} = \sqrt{6+x_n}$; 2) $x_1 = 1$, $x_{n+1} = \sqrt[3]{6+x_n}$;
3) $x_1 = 3$, $x_{n+1} = \sqrt{12+x_n}$, $n \in \mathbb{N}$.

Serilerin yakınsak olduğunu gösteriniz ve toplamını bulunuz:

$$\begin{array}{ll} 12.1. \sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}. & 12.2. \sum_{n=2}^{\infty} \frac{2}{4n^2-9}. \\ 12.3. \sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}. & 12.4. \sum_{n=3}^{\infty} \frac{2n+1}{n(n^2-1)}. \\ 12.5^*. \sum_{n=1}^{\infty} \frac{\cos in}{3^n}. & 12.6. \sum_{n=0}^{\infty} \frac{(1+i)^n}{2^n}. \end{array}$$

Yakınsaklığın gerekli şartını veya Cauchy kriterini kullanarak serilerin ıraksak olduğunu gösteriniz:

$$\begin{array}{ll} 12.7. \sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}. & 12.8. \sum_{n=1}^{\infty} \frac{n}{n^2+2}. \\ 12.9. \sum_{n=1}^{\infty} (-1)^n. & 12.10. \sum_{n=1}^{\infty} \frac{e^n}{n^{10}}. \end{array}$$

Karşılaştırma testini veya bu testin limit formunu kullanarak serilerin yakınsaklığını inceleyiniz:

$$\begin{array}{ll} 12.19. \sum_{n=1}^{\infty} \frac{1}{3n-2}. & 12.20. \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}. \\ 12.21. \sum_{n=1}^{\infty} \frac{n}{3n^3-1}. & 12.22. \sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)(n+2)}}. \\ 12.23. \sum_{n=1}^{\infty} \sin \frac{\pi}{n}. & 12.24. \sum_{n=1}^{\infty} \frac{n^2+3}{4n^3+5n}. \\ 12.25. \sum_{n=3}^{\infty} n^2 \operatorname{tg}^6 \frac{\pi}{n}. & 12.26. \sum_{n=1}^{\infty} \sqrt[3]{n^2} \operatorname{arctg} \frac{1}{n^2}. \\ 12.27. \sum_{n=2}^{\infty} \frac{\ln n}{\sqrt[3]{n^2}}. & 12.28. \sum_{n=1}^{\infty} e^{-n^2}. \\ 12.29. \sum_{n=1}^{\infty} \frac{2^n}{(n+1)3^n}. & 12.30. \sum_{n=1}^{\infty} \frac{\cos \sqrt{n} + i \sin \sqrt{n}}{n^2}. \end{array}$$

D'Alembert oran testine göre serinin yakınsaklığını inceleyiniz:

$$\begin{array}{ll} 12.31. \sum_{n=1}^{\infty} \frac{n^2+5}{2^n}. & 12.32. \sum_{n=1}^{\infty} \frac{n^n}{n!}. \\ 12.33. \frac{3}{1} + \frac{3 \cdot 5}{1 \cdot 4} + \dots + \frac{3 \cdot 5 \dots (2n+1)}{1 \cdot 4 \dots (3n-2)} + \dots \end{array}$$

$$12.34. \sum_{n=1}^{\infty} \frac{n^5}{5^n}.$$

$$12.35. \sum_{n=1}^{\infty} \frac{(3n+1)!}{8^n n^2}.$$

$$12.36. \sum_{n=1}^{\infty} \frac{e^{2n+1}}{n!}.$$

$$12.37. \sum_{n=2}^{\infty} \frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{2^{2n} (n-1)!}.$$

D'Alambert oran testini kullanarak serinin yakınsaklığını inceleyiniz:

$$12.31. \sum_{n=1}^{\infty} \frac{n^2 + 5}{2^n}.$$

$$12.32. \sum_{n=1}^{\infty} \frac{n^n}{n!}.$$

$$12.33. \frac{3}{1} + \frac{3 \cdot 5}{1 \cdot 4} + \dots + \frac{3 \cdot 5 \dots (2n+1)}{1 \cdot 4 \dots (3n-2)} + \dots$$

$$12.34. \sum_{n=1}^{\infty} \frac{n^5}{5^n}.$$

$$12.35. \sum_{n=1}^{\infty} \frac{(3n+1)!}{8^n n^2}.$$

$$12.36. \sum_{n=1}^{\infty} \frac{e^{2n+1}}{n!}.$$

$$12.37. \sum_{n=2}^{\infty} \frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{2^{2n} (n-1)!}.$$

$$12.38. \sum_{n=1}^{\infty} \frac{\sin in}{3^n}.$$

$$12.39. \sum_{n=1}^{\infty} \frac{n^n}{n!(e-i)^n}.$$

Cauchy oran testini kullanarak serinin yakınsaklığını inceleyiniz:

$$12.40. \sum_{n=1}^{\infty} \left(\frac{n-1}{2n+1} \right)^n.$$

$$12.41. \sum_{n=1}^{\infty} \left(1 + \frac{1}{n} \right)^{n^2}.$$

$$12.42. \sum_{n=1}^{\infty} \frac{1}{2^n} \left(1 + \frac{1}{n} \right)^{n^2}.$$

$$12.43. \sum_{n=1}^{\infty} \frac{1}{3^n} \left(1 + \frac{1}{n} \right)^{n^2}.$$

$$12.44. \sum_{n=1}^{\infty} \sqrt{n} \left(\frac{n}{4n-3} \right)^{2n}.$$

$$12.45. \sum_{n=1}^{\infty} n \left(\arcsin \frac{1}{n} \right)^n.$$

$$12.46. \sum_{n=1}^{\infty} u_n, \text{ где } u_{2k-1} = \left(\frac{k}{2k+1} \right)^k, u_{2k} = \left(\frac{k}{3k-1} \right)^{k/2}.$$

$$12.47. \sum_{n=1}^{\infty} \left(\frac{n+2i}{(1+i)n+3} \right)^n.$$

$$12.48. \sum_{n=1}^{\infty} \left(\frac{i(2n+i)}{4n} \right)^n.$$

Cauchy integral testini kullanarak serinin yakınsaklığını inceleyiniz:

$$12.49. \sum_{n=2}^{\infty} \frac{1}{n \ln^2 n}.$$

$$12.50. \sum_{n=2}^{\infty} \frac{1}{n \sqrt{\ln n}}.$$

$$12.51. \sum_{n=2}^{\infty} \frac{1}{n \ln n}.$$

$$12.52. \sum_{n=3}^{\infty} \frac{1}{n \ln n (\ln \ln n)^2}.$$

Serilerin yakınsaklığını inceleyiniz:

$$12.53. \sum_{n=1}^{\infty} \left(\frac{1+n}{1+n^2} \right)^2. \quad 12.54. \sum_{n=2}^{\infty} \frac{1}{\sqrt{n}} \ln \frac{n+1}{n-1}.$$

$$12.55. \sum_{n=1}^{\infty} \frac{n^2}{3^n}. \quad 12.56. \sum_{n=1}^{\infty} \frac{1}{n!}.$$

$$12.57. \sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^n. \quad 12.58. \sum_{n=1}^{\infty} \frac{n}{n^3+1}.$$

$$12.59. \sum_{n=1}^{\infty} \frac{n^n}{(2n-1)^{n-1}}. \quad 12.60. \sum_{n=1}^{\infty} \left(1 - \frac{1}{n} \right)^{n^2}.$$

$$12.61. 100 + \frac{100 \cdot 103}{1 \cdot 5} + \dots + \frac{100 \cdot 103 \dots (97+3n)}{1 \cdot 5 \cdot 9 \dots (4n-3)} + \dots$$

$$12.62. 1 + \frac{1 \cdot 11}{3!} + \frac{1 \cdot 11 \cdot 21}{5!} + \dots + \frac{1 \cdot 11 \cdot 21 \dots (10n-9)}{(2n-1)!} + \dots$$

$$12.63. \frac{1}{2} + \frac{1 \cdot 5}{2 \cdot 4 \cdot 6} + \frac{1 \cdot 5 \cdot 9}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10} + \dots + \frac{1 \cdot 5 \cdot 9 \dots (4n-3)}{(4n-2)!!} + \dots$$

$$12.64. \sum_{n=2}^{\infty} \frac{\sqrt{n+1} - \sqrt{n-1}}{n^{3/4}}. \quad 12.65. \sum_{n=2}^{\infty} \left(\frac{n-1}{n+1} \right)^{n(n-1)}.$$

$$12.66. \sum_{n=1}^{\infty} \left(\ln \frac{1}{n^{2/5}} - \ln \left(\sin \frac{1}{n^{2/5}} \right) \right).$$

$$12.67. \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}. \quad 12.68. \sum_{n=1}^{\infty} \sin \frac{\pi}{2n^2}.$$

$$12.69. \sum_{n=1}^{\infty} \ln \left(1 + \frac{1}{n^2} \right). \quad 12.70. \sum_{n=3}^{\infty} \frac{1}{n \ln n \ln \ln n}.$$

$$12.71. \sum_{n=1}^{\infty} \frac{3^n \cdot n!}{n^n}. \quad 12.72^*. \sum_{n=1}^{\infty} \frac{e^n \cdot n!}{n^n}.$$

$$12.73. 2 + \frac{2 \cdot 5}{1 \cdot 5} + \frac{2 \cdot 5 \cdot 8}{1 \cdot 5 \cdot 9} + \dots + \frac{2 \cdot 5 \cdot 8 \dots (3n-1)}{1 \cdot 5 \cdot 9 \dots (4n-3)} + \dots$$

$$12.74. \sum_{n=1}^{\infty} \frac{n^{100}}{2^n}. \quad 12.75. \sum_{n=1}^{\infty} \ln \left(1 + \frac{1}{n} \right).$$

$$12.76. \sum_{n=1}^{\infty} \left(\frac{2n-1}{5n+3} \right)^n. \quad 12.77. \sum_{n=1}^{\infty} \left(\frac{n}{n+2} \right)^n.$$

$$12.78. \frac{1}{100} + \frac{1 \cdot 4}{100 \cdot 102} + \frac{1 \cdot 4 \cdot 7}{100 \cdot 102 \cdot 104} + \dots + \dots + \frac{1 \cdot 4 \cdot 7 \dots (3n-2)}{100 \cdot 102 \dots (98+2n)} + \dots$$