

ALİŞTIRMALAR

İntegraller tablosuna göre aşağıdaki integralleri bulunuz:

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|--|---|---|
| 1. $\int x^2 \sqrt[3]{x} dx.$ | 2. $\int \frac{dx}{\sqrt[4]{x^3}}.$ | 3. $\int \sqrt{x} \sqrt{x} \sqrt{x} dx.$ |
| 4. $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx.$ | 5. $\int 2^x 8^x dx.$ | 6. $\int \frac{6^x}{3^{2x}} dx.$ |
| 7. $\int \frac{2^x + 5^x}{10^x} dx.$ | 8. $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx.$ | 9. $\int \frac{\sin 2x}{\sin x \cos^3 x} dx.$ |
| 10. $\int \frac{\sin 7x + \sin 3x}{\sin 5x \cos 2x} dx.$ | 11. $\int (\operatorname{tg} x - \operatorname{ctg} x)^2 dx.$ | 12. $\int \frac{dx}{\cos^2 x \sin^2 x}.$ |
| 13. $\int \frac{dx}{\operatorname{sh}^2 x \operatorname{ch}^2 x}.$ | 14. $\int \frac{\operatorname{sh} 2x}{\operatorname{ch} x} dx.$ | 15. $\int \operatorname{th}^2 x dx.$ |
| 16. $\int \operatorname{cth}^2 x dx.$ | 17. $\int \frac{dx}{4x^2 + 9}.$ | 18. $\int \frac{dx}{4x^2 - 9}.$ |
| 19. $\int \frac{dx}{\sqrt{4x^2 + 9}}.$ | 20. $\int \frac{dx}{\sqrt{9 - 4x^2}}.$ | |

Değişken değiştirme yöntemiyle integralleri bulunuz:

- | | | |
|--|--|---|
| 21. $\int x e^{\frac{x^2}{2}} dx.$ | 22. $\int x^2 \sin \frac{x^3}{3} dx.$ | 23. $\int \frac{x}{\sqrt{1-x^2}} dx.$ |
| 24. $\int \frac{dx}{x \ln x}.$ | 25. $\int \frac{dx}{\sqrt[4]{x^3} (1 + \sqrt[4]{x})}.$ | 26. $\int \frac{dx}{\sqrt[4]{x^3} \sqrt{1 - \sqrt[4]{x}}}.$ |
| 27. $\int \frac{dx}{\sqrt{e^x + 1}}.$ | 28. $\int \frac{e^{2x}}{e^x - 1} dx.$ | 29. $\int x^2 (x-1)^{18} dx.$ |
| 30. $\int \frac{dx}{x \sqrt{x-1}}.$ | 31. $\int x \sqrt[3]{x+1} dx.$ | 32. $\int \frac{x}{\sqrt[3]{x+1}} dx.$ |
| 33. $\int \frac{x^3}{\sqrt{1-x^2}} dx.$ | 34. $\int \frac{x dx}{1+x^4}.$ | 35. $\int \frac{x^2}{\sqrt{1-x^6}} dx.$ |
| 36. $\int \frac{\ln \operatorname{tg} x}{\sin 2x} dx.$ | 37. $\int \frac{\operatorname{ctg} x}{\ln \sin x} dx.$ | 38. $\int \frac{\sin x}{\cos^3 x} e^{u^2 x} dx.$ |
| 39. $\int \frac{\arctg x \cdot e^{\arctg^2 x}}{1+x^2} dx.$ | 40. $\int \frac{\ln x}{x(4 + \ln^2 x)} dx.$ | |

Kısmi integrasyon yöntemiyle aşağıdaki integralleri bulunuz:

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|-----------------------------------|---|----------------------------------|
| 41. $\int x e^{-x} dx.$ | 42. $\int x 2^x dx.$ | 43. $\int x \sin 2x dx.$ |
| 44. $\int (1+x) e^x dx.$ | 45. $\int (1+x \ln 2) 2^x dx.$ | 46. $\int (2x - x^2) e^{-x} dx.$ |
| 47. $\int \arctg x dx.$ | 48. $\int x \operatorname{arccotg} x dx.$ | 49. $\int \arcsin x dx.$ |
| 50. $\int \frac{x}{\cos^2 x} dx.$ | 51. $\int x \operatorname{tg}^2 x dx.$ | 52. $\int \cos(\ln x) dx.$ |
| 53. $\int \sin(\ln x) dx.$ | | |

Basit kesirlerin integrallerini bulunuz:

$$\begin{array}{lll}
 54. \int \frac{2 dx}{5+2x} & 55. \int \frac{dx}{2-3x} & 56. \int \frac{dx}{(3x+5)^3} \\
 57. \int \frac{dx}{(1-2x)^3} & 58. \int \frac{dx}{x^2+2x+3} & 59. \int \frac{x+2}{x^2+2x+5} dx \\
 60. \int \frac{x+1}{x^2-x+2} dx & 61. \int \frac{x dx}{x^2+7x+13} &
 \end{array}$$

Belirsiz katsayılar yöntemiyle rasyonel kesirlerin integrallerini bulunuz:

$$\begin{array}{lll}
 62. \int \frac{2x+3}{x^2+3x-10} dx & 63. \int \frac{x dx}{(x+1)(2x+1)} & 64. \int \frac{3x^2-2x-4}{(x-1)(x^2-4)} dx \\
 65. \int \frac{x^3-x+2}{x^2-1} dx & 66. \int \frac{x^4+1}{x^3-x} dx & 67. \int \frac{x+2}{(x+3)^2} dx \\
 68. \int \frac{x^2+2x}{(x+1)^4} dx & 69. \int \frac{dx}{x^3+x^4} & 70. \int \frac{x^2-3x}{(x+1)(x-1)^2} dx \\
 71. \int \frac{dx}{x(x^2+1)} & 72. \int \frac{dx}{x^4-1} & 73. \int \frac{x dx}{x^3-1}
 \end{array}$$

İrrasyonel fonksiyonların integrallerini bulunuz:

$$\begin{array}{lll}
 74. \int \frac{1}{(1-x)^2} \cdot \sqrt{\frac{1-x}{1+x}} dx & 75. \int \frac{\sqrt{1+x}}{x} dx & 76. \int \frac{1}{(1-x)(1+x)^2} \cdot \sqrt{\frac{1+x}{1-x}} dx \\
 77. \int \sqrt[3]{\frac{1-x}{1+x}} \cdot \frac{dx}{(1+x)^2} & 78. \int \frac{dx}{\sqrt{x^2-4x}} & 79. \int \frac{dx}{\sqrt{4x-x^2}} \\
 80. \int \frac{dx}{\sqrt{x^2-4x-5}} & 81. \int \frac{dx}{\sqrt{x^2-4x+5}} & 82. \int \frac{dx}{\sqrt{5-4x-x^2}} \\
 83. \int \frac{x+1}{\sqrt{x^2-6x-1}} dx & 84. \int \frac{x+1}{\sqrt{1+6x-x^2}} dx & 85. \int \frac{x-3}{\sqrt{1+6x-x^2}} dx \\
 86. \int \frac{2x^2+3x+2}{\sqrt{x^2+2x+2}} dx & 87. \int \frac{1+2x-3x^3}{\sqrt{1-x^2}} dx & 88. \int \frac{2x^2+1}{\sqrt{x^2+2}} dx \\
 89. \int \frac{x^5 dx}{\sqrt{1-x^2}} & 90. \int \frac{x^6}{\sqrt{1+x^2}} dx &
 \end{array}$$

Trigonometrik fonksiyonların integrallerini bulunuz:

$$\begin{array}{lll}
 91. \int \frac{dx}{2+\cos x} & 92. \int \frac{dx}{5+4 \sin x} & 93. \int \frac{dx}{3 \sin x-4 \cos x} \\
 94. \int \frac{dx}{5+\sin x+3 \cos x} & 95. \int \frac{\sin 2x}{1+\sin^2 x} dx & 96. \int \frac{\cos x dx}{5+\sin^2 x-6 \sin x} \\
 97. \int \frac{1+\cos x-\sin x}{1-\cos x+\sin x} dx & 98. \int \cos^3 x dx & 99. \int \sin^5 x dx \\
 100. \int \sin^2 x \cos^3 x dx & 101. \int \cos^4 x \sin^3 x dx & 102. \int \frac{\sin^3 x}{\cos^4 x} dx \\
 103. \int \sin^2 x \cos^2 x dx & 104. \int \sin^4 x dx & 105. \int \sin^4 x \cos^2 x dx
 \end{array}$$

$$\begin{array}{lll}
106. \int \frac{dx}{\sin^4 x} & 107. \int \frac{\cos^2 x}{\sin^4 x} dx & 108. \int \frac{dx}{\sin^2 x \cos^4 x} \\
109. \int \frac{dx}{\cos^4 x - \sin^4 x} & 110. \int \sin 5x \cos x dx & 111. \int \sin x \cos 5x dx \\
112. \int \cos 7x \cos 3x dx & 113. \int \sin 15x \sin 10x dx & 114. \int \cos \frac{x}{2} \cos \frac{x}{3} dx \\
115. \int \sin x \sin 2x \sin 3x dx
\end{array}$$

Cevaplar:

$$\begin{array}{l}
1. 0, 3x^{10/3} + C. 2. 4\sqrt[4]{x} + C. 3. \frac{8}{15}x^{15/8} + C. 4. \frac{x^2}{2} + 2x + \ln|x| + C. 5. \frac{16x}{\ln 16} + C. 6. \frac{1}{\ln \frac{2}{3}} \left(\frac{2}{3}\right)^x + C. \\
7. -\frac{5-x}{\ln 5} - \frac{x}{\ln 2} + C. 8. -\operatorname{ctg} x - \operatorname{tg} x + C. 9. 2 \operatorname{tg} x + C. 10. 2x + C. 11. \operatorname{tg} x - \operatorname{ctg} x - 4x + C. \\
12. \operatorname{tg} x - \operatorname{ctg} x + C. 13. -\operatorname{th} x - \operatorname{cth} x + C. 14. 2 \operatorname{ch} x + C. 15. x - \operatorname{th} x + C. 16. x - \operatorname{ch} x + C. \\
17. \frac{1}{6} \operatorname{tg}^{-1} \frac{2}{3} x + C. 18. \frac{1}{12} \ln \left| \frac{2x-3}{2x+3} \right| + C. 19. \ln \sqrt{2x + \sqrt{4x^2 + 9}} + C. 20. \frac{1}{2} \sin^{-1} \frac{2}{3} x + C. 21. e^{x^2/2} + C; \\
22. -\cos \frac{x}{3} + C. 23. -\sqrt{1-x^2} + C. 24. \ln|\ln x| + C. 25. 4 \ln(1 + \sqrt[4]{x}) + C. 26. -8\sqrt{1-\sqrt[4]{x}} + C. \\
27. -2 \ln(e^{-x/2} + \sqrt{1+e^{-x}}) + C. 28. \ln|e^x - 1| + e^x + C. 29. \frac{(x-1)^{21}}{21} + \frac{(x-1)^{20}}{10} + \frac{(x-1)^{19}}{19} + C. \\
30. 2 \operatorname{tg}^{-1} \sqrt{x-1}. 31. \frac{3}{7}(x+1)^{7/3} - \frac{3}{4}(x+1)^{4/3} + C. 32. \frac{3}{10}(x+1)^{2/3}(2x-3) + C. 33. -\sqrt{1-x^2} + \\
\frac{1}{3} \sqrt{(1-x)^3} + C. 34. \frac{1}{2} \operatorname{tg}^{-1} x^2 + C. 35. \frac{1}{3} \sin^{-1} x^3 + C. 36. \frac{1}{4} \ln^2 \operatorname{tg} x + C. 37. \ln|\ln \sin x| + C. \\
38. \frac{1}{2} e^{x^2} + C. 39. \frac{1}{2} e^{\operatorname{arctg}^2 x} + C. 40. \ln \sqrt{4 + \ln^2 x} + C. 41. -(x+1)e^{-x} + C. 42. \frac{x \ln 2 - 1}{\ln^2 2} 2^x + C. \\
43. \frac{1}{4} \sin 2x - \frac{\pi}{2} \cos 2x + C. 44. xe^x + C. 45. x^2 e^x + C. 46. x^2 e^{-x} + C. 47. x \operatorname{arctg} x - \ln \sqrt{1+x^2} + C. \\
48. \frac{1}{2}(x^2 + 1) \operatorname{arctg} x - \frac{1}{2} x + C. 49. x \sin^{-1} x + \sqrt{1-x^2} + C. 50. x \operatorname{tg} x + \ln|\cos x| + C. \\
51. -\frac{x^2}{2} + x \operatorname{tg} x + \ln|\cos x| + C. 52. \frac{\pi}{2}(\cos \ln x + \sin \ln x) + C. 53. \frac{\pi}{2}(\sin \ln x - \cos \ln x) + C. \\
54. \ln|5+2x| + C. 55. -\frac{1}{3} \ln|2-3x| + C. 56. -\frac{1}{6(3x+5)^2} + C. 57. \frac{1}{8(1-2x)^4} + C. 58. \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{x+1}{\sqrt{2}} + C. \\
59. \ln \sqrt{x^2+2x+5} + \frac{1}{2} \operatorname{tg} \frac{x+1}{2} + C. 60. \ln \sqrt{x^2-x+2} + \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{2x-1}{\sqrt{2}} + C. 61. \ln \sqrt{x^2+7x+13} - \\
\frac{1}{\sqrt{3}} \operatorname{arctg} \frac{2x+7}{\sqrt{3}} + C. 62. \ln|x^2+3x-10| + C. 63. \ln \left| \frac{x+1}{\sqrt{2x+1}} \right| + C. 64. \ln|(x-1)(x^2-4)| + C. \\
65. \frac{x^2}{2} + \ln \left| \frac{x-1}{x+1} \right| + C. 66. \frac{x^2}{2} + \ln \left| \frac{x^2-1}{x} \right| + C. 67. \frac{1}{x+3} + \ln|x+3| + C. 68. \frac{1}{3(x+1)^3} - \frac{1}{x+1} + C. \\
69. \frac{1}{x} - \frac{1}{2x^2} + \ln \left| \frac{x}{x+1} \right| + C. 70. \frac{1}{x-1} + \ln|x+1| + C. 71. \ln \frac{|x|}{\sqrt{x^2+1}} + C. 72. \frac{1}{4} \ln \left| \frac{x-1}{x+1} \right| - \frac{1}{2} \operatorname{arctg} x + C. \\
73. \frac{1}{3} \left(\ln \left| \frac{x-1}{\sqrt{x^2+x+1}} \right| + \sqrt{3} \operatorname{arctg} \frac{2x+1}{\sqrt{3}} \right) + C. 74. \sqrt{\frac{1+x}{1-x}} + C. 75. 2\sqrt{1+x} + \ln \left| \frac{\sqrt{1+x}-1}{\sqrt{1+x}+1} \right| + C. 76. \frac{x}{\sqrt{1-x^2}} + C. \\
77. -\frac{3}{8} \left(\frac{1-x}{1+x} \right)^{4/3} + C. 78. \ln|x-2+\sqrt{x^2-4x}| + C. 79. \arcsin \frac{x-2}{2} + C. 80. \ln|x-2+\sqrt{x^2-4x-5}| + C. \\
81. \ln(x-2+\sqrt{x^2-4x+5}) + C. 82. \arcsin \frac{x+2}{3}. 83. \sqrt{x^2-6x-1} + 4 \ln|x-3+\sqrt{x^2-6x-1}| + C. \\
84. 2\sqrt{1+6x-x^2} + 4 \arcsin \frac{x-3}{\sqrt{10}} + C. 85. -\sqrt{1+6x-x^2} + C. 86. x\sqrt{x^2+2x+2} + C. \\
87. x^2\sqrt{1-x^2} + \arcsin x + C. 88. x\sqrt{x^2+2} - \ln(x+\sqrt{x^2+2}) + C. 89. -\frac{8+4x^2+3x^4}{15}\sqrt{1-x^2} + C. \\
90. \left(\frac{5}{16}x - \frac{5}{24}x^3 + \frac{1}{6}x^5 \right) \sqrt{1+x^2} - \frac{5}{16} \ln(x+\sqrt{x^2+1}) + C. 91. \ln \left(3 + \operatorname{tg}^2 \frac{\pi}{2} \right) + C. 92. \frac{2}{3} \operatorname{tg}^{-1} \frac{5 \operatorname{tg} \frac{\pi}{2} + 4}{3} + C. \\
93. \frac{1}{3} \ln \left| \frac{2 \operatorname{tg} \frac{\pi}{2} - 1}{2 \operatorname{tg} \frac{\pi}{2} + 1} \right| + C. 94. \frac{2}{\sqrt{15}} \operatorname{arctg} \frac{2 \operatorname{tg} \frac{\pi}{2} + 1}{\sqrt{15}} + C. 95. \ln(1 + \sin^2 x) + C. 96. \frac{1}{4} \ln \frac{5 - \sin x}{1 - \sin x} + C. \\
97. -x + 2 \ln \left| \frac{\operatorname{tg} \frac{\pi}{2}}{1 + \operatorname{tg} \frac{\pi}{2}} \right| + C. 98. \sin x - \frac{1}{3} \sin^3 x + C. 99. -\frac{1}{2} \cos^5 x + \frac{2}{3} \cos^3 x - \cos x + C. 100. \frac{1}{3} \sin^3 x - \\
\frac{1}{5} \sin^5 x + C. 101. \frac{1}{7} \cos^7 x - \frac{1}{5} \cos^5 x + C. 102. \frac{1}{3 \cos^3 x} - \frac{1}{\cos x} + C. 103. \frac{\pi}{8} - \frac{\sin 4x}{32} + C. 104. \frac{3}{8}x - \frac{1}{4} \sin 2x + \\
\frac{1}{32} \sin 4x + C. 105. \frac{\pi}{16} - \frac{\sin 4x}{64} - \frac{\sin^3 2x}{48} + C. 106. -\operatorname{ctg} x - \frac{1}{3} \operatorname{ctg}^3 x + C. 107. -\frac{1}{3} \operatorname{ctg}^3 x + C. 108. \operatorname{tg} x + \\
\frac{1}{3} \operatorname{tg}^3 x - 2 \operatorname{ctg} 2x + C. 109. -\frac{1}{2} \ln \left| \operatorname{tg} \left(x - \frac{\pi}{4} \right) \right| + C. 110. -\frac{\cos 6x}{12} - \frac{\cos 4x}{8} + C. 111. -\frac{\cos 6x}{12} + \frac{\cos 4x}{8} + C. \\
112. \frac{\sin 10x}{20} + \frac{\sin 4x}{8} + C. 113. -\frac{\sin 25x}{50} + \frac{\sin 5x}{10} + C. 114. \frac{3}{5} \sin \frac{5}{6}x + 3 \sin \frac{\pi}{6} + C. 115. \frac{\cos 6x}{24} - \frac{\cos 4x}{16} - \frac{\cos 2x}{8} + C.
\end{array}$$

$$1970. \int \frac{x dx}{\sqrt{3x^2 - 11x + 2}}.$$

$$1972. \int \frac{dx}{x \sqrt{x^2 + 2x - 1}}.$$

$$1974. \int \frac{\sqrt{x^2 + 2x}}{x} dx.$$

$$1976. \int \frac{dx}{x \sqrt{a^2 - x^2}}.$$

$$1978. \int \frac{dx}{x - \sqrt{x^2 - x + 1}}.$$

$$1980. \int \frac{dx}{(x + \sqrt{x^2 - 1})^2}.$$

$$1982. \int \frac{\sqrt{1 + x^2}}{2 + x^2} dx.$$

$$1984. \int \frac{(x - 1) dx}{x^2 \sqrt{2x^2 - 2x + 1}}.$$

$$1986. \int \frac{(2x + 3) dx}{(x^2 + 2x + 3) \sqrt{x^2 + 2x + 4}}.$$

$$1971. \int \frac{(8x - 11) dx}{\sqrt{5 + 2x - x^2}}.$$

$$1973. \int \frac{dx}{x \sqrt{2 + x - x^2}}.$$

$$1975. \int \frac{dx}{x^2 \sqrt{x^2 + a^2}}.$$

$$1977. \int \frac{dx}{(x^2 + 3)^{\frac{5}{2}}}.$$

$$1979. \int \frac{dx}{x^2 (x + \sqrt{1 + x^2})}.$$

$$1981. \int \frac{x^2 - x + 1}{\sqrt{(x^2 + 1)^3}} dx.$$

$$1983. \int \frac{dx}{1 + \sqrt{x^2 + 2x + 2}}.$$

$$1985. \int \frac{(2x^2 - 3x) dx}{\sqrt{x^2 - 2x + 5}}.$$