

Analiz III. Bazı Önemli Limitler

1. $\lim_{n \rightarrow \infty} a^n = +\infty, (a > 1); \quad \lim_{n \rightarrow \infty} a^n = 0, (0 < a < 1);$
2. $\lim_{n \rightarrow \infty} \log_a n = +\infty, (a > 1); \quad \lim_{n \rightarrow \infty} \log_a n = -\infty, (0 < a < 1);$
 $\lim_{n \rightarrow \infty} \log_a \frac{1}{n} = -\infty, (a > 1); \quad \lim_{n \rightarrow \infty} \log_a \frac{1}{n} = +\infty, (0 < a < 1);$
3. $\lim_{n \rightarrow \infty} \frac{a^n}{n^k} = +\infty, (k > 0, a > 1); \quad \lim_{n \rightarrow \infty} \frac{n^k}{a^n} = 0, (a > 1);$
 $\lim_{n \rightarrow \infty} n^k a^n = 0, (k > 0, 0 < a < 1);$
4. $\lim_{n \rightarrow \infty} \frac{\log_a n}{n^k} = 0, (k > 0, 0 < a \neq 1); \quad \lim_{n \rightarrow \infty} \frac{1}{n^k} \log_a \frac{1}{n} = 0, (0 < a \neq 1);$
5. $\lim_{n \rightarrow \infty} \sqrt[n]{a} = 1, (a > 0); \quad 6. \lim_{n \rightarrow \infty} \sqrt[n]{n} = 1; \quad 7. \lim_{n \rightarrow \infty} \frac{a^n}{n!} = 0, (a \in \mathbb{R})$
5. $\lim_{n \rightarrow \infty} x_n = a, x_n \neq a$ ve $\lim_{x \rightarrow a} f(x) = L \Rightarrow \lim_{n \rightarrow \infty} f(x_n) = L$ dir.
6. $\lim_{n \rightarrow \infty} x_n = a \Rightarrow \lim_{x \rightarrow a} \sqrt[m]{(x_n)^k} = \sqrt[m]{a^k}, (a \in \mathbb{R}, m, k \in \mathbb{N}^+, \text{ aralarında asal}$
ve m tek sayı ise yahut $a > 0$ ve m çift sayı ise) dir.
7. $\lim_{n \rightarrow \infty} n \sin \frac{1}{n} = \lim_{n \rightarrow \infty} n \tan \frac{1}{n} = \lim_{n \rightarrow \infty} n \arcsin \frac{1}{n} = \lim_{n \rightarrow \infty} n \arctan \frac{1}{n} = 1; \Leftrightarrow$
 $\sin \frac{1}{n} \sim \tan \frac{1}{n} \sim \arcsin \frac{1}{n} \sim \arctan \frac{1}{n} \sim \frac{1}{n}, n \rightarrow \infty;$
8. $\lim_{n \rightarrow \infty} n^2 \left(1 - \cos \frac{1}{n}\right) = \frac{1}{2}; \Leftrightarrow 1 - \cos \frac{1}{n} \sim \frac{1}{2n^2}, n \rightarrow \infty; \Leftrightarrow \cos \frac{1}{n} = 1 -$
 $\frac{1}{2n^2} + o\left(\frac{1}{n^2}\right), n \rightarrow \infty;$
9. $\lim_{n \rightarrow \pm \infty} \left(1 + \frac{x}{n}\right)^n = e^x, x \in \mathbb{R};$
10. $\lim_{n \rightarrow \infty} n (\sqrt[n]{a} - 1) = \ln a, (a > 0); \Leftrightarrow \sqrt[n]{a} - 1 \sim \frac{1}{n} \ln a, n \rightarrow \infty; \Leftrightarrow \sqrt[n]{a} =$
 $1 + \frac{\ln a}{n} + o\left(\frac{1}{n}\right), n \rightarrow \infty;$
11. $\lim_{x \rightarrow 0} n (\sqrt[n]{e} - 1) = 1; \Leftrightarrow \sqrt[n]{e} - 1 \sim \frac{1}{n}, n \rightarrow \infty; \Leftrightarrow \sqrt[n]{e} = 1 + \frac{1}{n} + o\left(\frac{1}{n}\right),$
 $n \rightarrow \infty;$
12. $\lim_{n \rightarrow \infty} n \log_a \left(1 + \frac{1}{n}\right) = \frac{1}{\ln a} = \log_a e, (0 < a \neq 1); \Leftrightarrow \log_a \left(1 + \frac{1}{n}\right) \sim \frac{1}{n \ln a},$
 $n \rightarrow \infty; \Leftrightarrow \log_a \left(1 + \frac{1}{n}\right) = \frac{1}{n \ln a} + o\left(\frac{1}{n}\right), n \rightarrow \infty;$
13. $\lim_{n \rightarrow +\infty} n \ln \left(1 + \frac{1}{n}\right) = 1; \Leftrightarrow \ln \left(1 + \frac{1}{n}\right) \sim \frac{1}{n}, n \rightarrow \infty; \Leftrightarrow \ln \left(1 + \frac{1}{n}\right) = \frac{1}{n} + o\left(\frac{1}{n}\right),$
 $n \rightarrow \infty;$
14. $\lim_{n \rightarrow \infty} \frac{(1 + \frac{1}{n})^\alpha - 1}{\frac{1}{n}} = \alpha, (\alpha \in \mathbb{R}); \Leftrightarrow (1 + \frac{1}{n})^\alpha - 1 \sim \frac{\alpha}{n}, n \rightarrow \infty; \Leftrightarrow (1 + \frac{1}{n})^\alpha =$
 $1 + \frac{\alpha}{n} + o\left(\frac{1}{n}\right), n \rightarrow \infty;$
15. $\lim_{n \rightarrow \infty} x_n = 0, x_n \neq 0, n \in \mathbb{N}^+ \Rightarrow \lim_{n \rightarrow \infty} \frac{(1 + x_n)^\alpha - 1}{x_n} = \alpha, (\alpha \in \mathbb{R});$