

Analiz I. Bazı Önemli Limitler

1. $\lim_{x \rightarrow +\infty} a^x = +\infty, (a > 1); \quad \lim_{x \rightarrow -\infty} a^x = 0, (a > 1);$
 $\lim_{x \rightarrow -\infty} a^x = +\infty, (0 < a < 1); \quad \lim_{x \rightarrow +\infty} a^x = 0, (0 < a < 1);$
2. $\lim_{x \rightarrow +\infty} \log_a x = +\infty, (a > 1); \quad \lim_{x \rightarrow 0^+} \log_a x = -\infty, (a > 1);$
 $\lim_{x \rightarrow +\infty} \log_a x = -\infty, (0 < a < 1); \quad \lim_{x \rightarrow 0^+} \log_a x = +\infty, (0 < a < 1);$
3. $\lim_{x \rightarrow +\infty} \frac{a^x}{x^k} = +\infty, (k > 0, a > 1); \quad \lim_{x \rightarrow +\infty} \frac{x^k}{a^x} = 0, (a > 1);$
 $\lim_{x \rightarrow +\infty} x^k a^x = 0, (k > 0, 0 < a < 1); \quad \lim_{x \rightarrow -\infty} |x|^k a^x = 0, (k > 0, a > 1);$
4. $\lim_{x \rightarrow +\infty} \frac{\log_a x}{x^k} = 0, (k > 0, 0 < a \neq 1); \quad \lim_{x \rightarrow 0^+} x^k \log_a x = 0, (0 < a \neq 1);$
5. $\lim_{x \rightarrow a} f(x) = L \Rightarrow \lim_{x \rightarrow a} \sqrt[m]{[f(x)]^k} = \sqrt[m]{L^k}, (L \in \mathbb{R}, m, k \in \mathbb{N}^+, \text{ aralarında}$
asal ve m tek sayı ise yahut $L > 0$ ve m çift sayı ise). Burada $a \in \mathbb{R}$ veya $a = +\infty, -\infty, \infty$ dır.
6. $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1; \Leftrightarrow \sin x \sim x, x \rightarrow 0; \Leftrightarrow \sin x = x + o(x), x \rightarrow 0;$
7. $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1; \Leftrightarrow \tan x \sim x, x \rightarrow 0; \Leftrightarrow \tan x = x + o(x), x \rightarrow 0;$
8. $\lim_{x \rightarrow 0} \frac{\arcsin x}{x} = 1; \Leftrightarrow \arcsin x \sim x, x \rightarrow 0; \Leftrightarrow \arcsin x = x + o(x), x \rightarrow 0;$
9. $\lim_{x \rightarrow 0} \frac{\arctan x}{x} = 1; \Leftrightarrow \arctan x \sim x, x \rightarrow 0; \Leftrightarrow \arctan x = x + o(x), x \rightarrow 0;$
10. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}; \Leftrightarrow 1 - \cos x \sim \frac{x^2}{2}, x \rightarrow 0; \Leftrightarrow 1 - \cos x = \frac{x^2}{2} + o(x^2),$
 $x \rightarrow 0;$
11. $\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x}\right)^x = e;$ 12. $\lim_{x \rightarrow -\infty} \left(1 + \frac{1}{x}\right)^x = e;$ 13. $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e;$
14. $\lim_{t \rightarrow 0} (1 + t)^{\frac{1}{t}} = e;$
15. $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a; \Leftrightarrow a^x - 1 \sim x \ln a, x \rightarrow 0; \Leftrightarrow a^x = 1 + x \ln a + o(x),$
 $x \rightarrow 0;$
16. $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1; \Leftrightarrow e^x - 1 \sim x, x \rightarrow 0; \Leftrightarrow e^x = 1 + x + o(x), x \rightarrow 0;$
17. $\lim_{x \rightarrow 0} \frac{\log_a(1+x)}{x} = \frac{1}{\ln a} = \log_a e, (0 < a \neq 1); \Leftrightarrow \log_a(1+x) \sim \frac{x}{\ln a},$
 $x \rightarrow 0; \Leftrightarrow \log_a(1+x) = \frac{x}{\ln a} + o(x), x \rightarrow 0;$
18. $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1; \Leftrightarrow \ln(1+x) \sim x, x \rightarrow 0; \Leftrightarrow \ln(1+x) = x + o(x),$
 $x \rightarrow 0;$
19. $\lim_{x \rightarrow 0} \frac{(1+x)^\alpha - 1}{x} = \alpha, (\alpha \in \mathbb{R}); \Leftrightarrow (1+x)^\alpha - 1 \sim \alpha x, x \rightarrow 0; \Leftrightarrow (1+x)^\alpha =$
 $1 + \alpha x + o(x), x \rightarrow 0;$